PARAMETERIZATION OF TERRAIN IN ARMY COMBAT ANALYSIS

Christopher James Needels

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

PARAMETERIZATION OF TERRAIN
IN ARMY COMBAT ANALYSIS

Christopher James Needels

March 1976

Thesis Advisor: Samuel H. Parry

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) Parameterization of Terrain in Army Combat Analysis	5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; March 1976
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Christopher James Needels	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	12. REPORT DATE March 1976
	13. NUMBER OF PAGES 87
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Naval Postgraduate School Monterey, California 93940	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

1. Terrain Modelling
2. SIMTER (Simulated Terrain Model)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This study presents and evaluates a methodology for parameterizing terrain for use in land combat analysis. The current procedure is to use digitized data which is compiled from actual terrain by engineer surveys and photo-interpretation. However, for those studies which do not require exact representation of terrain, a less costly and time consuming method can be used. In particular, terrain can be created

mathematically by using a modified bivariate normal probability density function. An additional advantage of this approach is that the macro-terrain features can be created at random, thereby providing multiple, unique realizations of a type of terrain. This capability overcomes the sensitivity of Army study results to a single sample of terrain. When used for line-of-sight calculations, the parameterized, continuous representation eliminates the need for interpolations required for digitized terrain. The methodology and simulation can be employed independently or used as a preprocessor for other combat models.

Parameterization Of Terrain
in Army Combat Analysis

by

Christopher James Needels
Major, United States Army
B.S., United States Military Academy, 1965

Submitted in partial fulfillment of the
requirement for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the
NAVAL POSTGRADUATE SCHOOL
March 1976

ABSTRACT

This study presents and evaluates a methodology for parameterizing terrain for use in land combat analysis. The current procedure is to use digitized data which is compiled from actual terrain by engineer surveys and photo-interpretation. However, for those studies which do not require exact representation of terrain, a less costly and time consuming method can be used. In particular, terrain can be created mathematically by using a modified bivariate normal probability density function. An additional advantage of this approach is that the macro-terrain features can be created at random, thereby providing multiple, unique realizations of a type of terrain. This capability overcomes the sensitivity of Army study results to a single sample of terrain. When used for line-of-sight calculations, the parameterized, continous representation eliminates the need for interpolations required for digitized terrain. The methodology and simulation can be employed independently or used as a preprocessor for other combat models.

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

The representation of terrain in many Army combat simulation models is both costly and time consuming. For instance, terrain input for high resolution models such as Dynamic Tactical Simulation (DYNTACS), Tank Exchange Model (TXM), and various versions of the Bonder/Individual Unit Action model comes from computer tapes prepared by such agencies as the Army Map Service and the Waterways Experiment Station. These tapes typically contain terrain data at ten meter intervals for an entire piece of actual terrain. To survey and record the 18,000 grid intersections for a six kilometer by three kilometer section of terrain is expensive and, as is suggested by this paper, unnecessary for many applications. The storage of this information not only can require a major portion of a computers storage capacity (core), but also require a great deal of running time searching this stored data in order to make movement and line-of-sight calculations. The exact amount of core and running time depends upon the type computer, the resolution of the model, and the dimensions of the selected peice of terrain.

The current method of terrain representation is also statistically questionable. In a Vector Research, Incorporated (VRI) study on terrain line-of-sight, it concluded that "...present and past Army study results, based on the analysis of combat results on a very limited sample of terrains, may have been determined by the terrain selection process and not by the actual weapon system or force design differences." More specifically it stated the following:

a. "There is extreme sensitivty in combat model results as the scenarios (terrain and movement assumptions) are varied, even when variation is within a class of scenarios chosen for their "a priori" equivalence.

b. "This sensitivity can be slightly reduced, but remains extreme (with probabilities of win estimable only within plus or minus 25%) even when battle results are used to redesign scenarios."

These results imply that sufficient replications of each type of terrain should be run in order to reach a satisfactory statistical level of significance. For VRI's analysis at least 50 replications of each type of terrain were used.

The problems stated above suggest that new methodology for the representation of terrain is necessary in order to reduce costs, shorten computer running time, and improve the level of significance for Army studies which use combat models. Consequently, the objective of this paper is to develop this new technology using parameterized, randomly created terrain, and to verify the concepts using a computerized terrain simulation model.

---

1. Farrell, Robert L., Freedman, Richard J., *Investigations of the Variation of Combat Model Predictions with Terrain Line of Sight*, p. 7, Vector Research, Incorporated, 1975.

2. *Ibid.*, p. 6.

3. *Ibid.*, p. 5.

II. TERRAIN REPRESENTATION

A. STATE OF THE ART

DYNTACS, which is one of the most frequently used high resolution combat simulations, serves as a good example of the state of the art in terrain representation and use. In general terms DYNTACS is a two-sided, dynamic, Monte Carlo, highly interactive combat simulation capable of representing forces from a single crew served weapon to a reinforced battalion. In particular the model considers in detail the effects of terrain on detection, mobility, tactics, and intervisibility between weapon elements. For the remainder of this paper, unless otherwise specified, only macro-terrain (elevation and slope) will be considered. As previously mentioned this data is supplied by the Waterways Experiment Station (WES) on computer tapes. Although these tapes have a ten meter resolution, DYNTACS is generally run using 100 meter squares. The simulation divides each grid square diagonally, thus producing a series of adjoining triangular terrains. The entire battlefield, therefore, is represented as a surface of equally sized triangles which vary in slope depending upon the elevation at their corners.

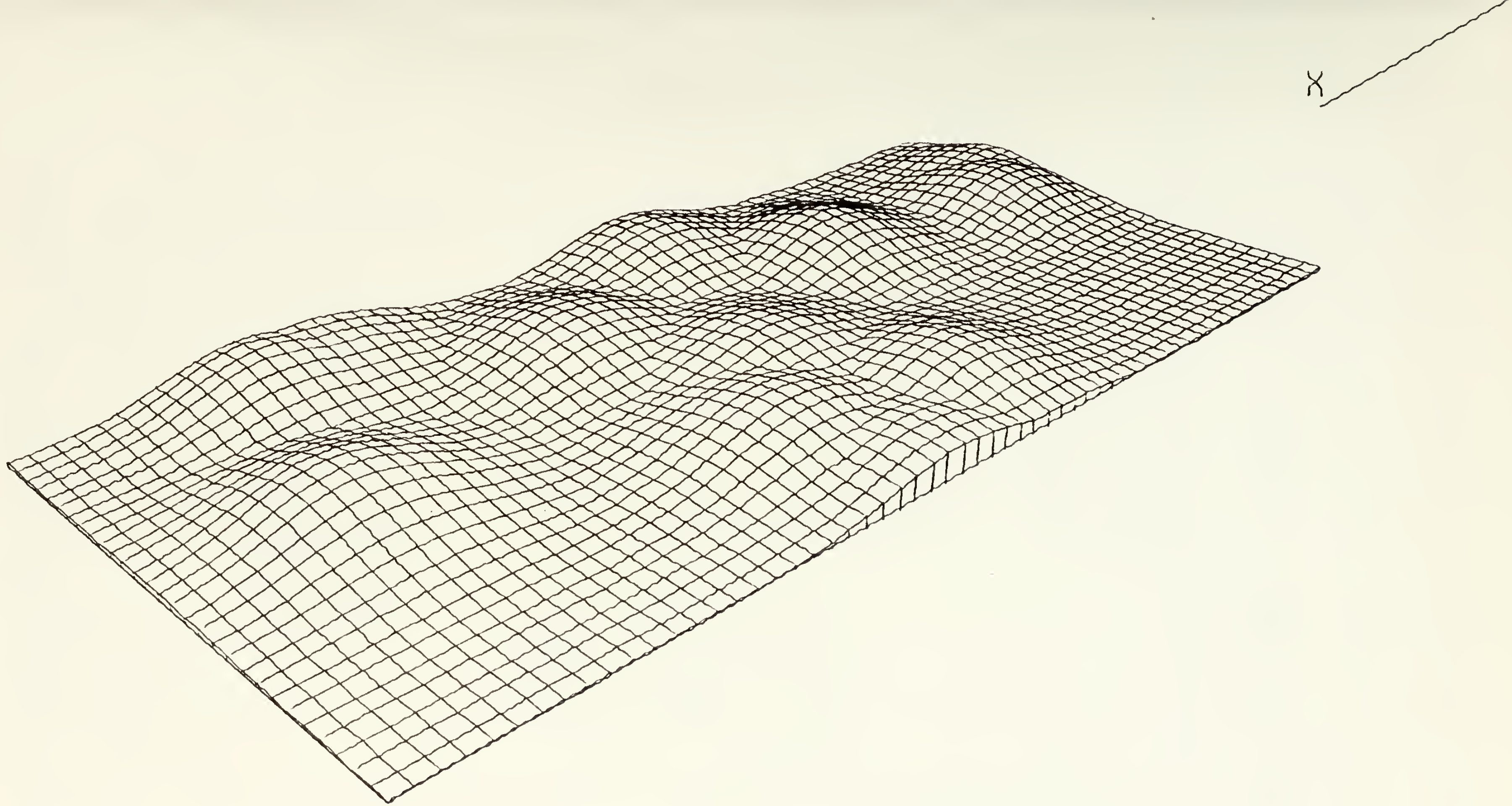
From the macro-terrain data, line-of-sight between any two opposing elements is computed. This is accomplished by first computing an angle between the horizontal and a straight line drawn between an observer and target (O-T line). The program then conducts a search of the terrain along the path of the O-T line to see if any macro-terrain is higher than the O-T line itself. This is accomplished by comparing the angle of the O-T line with the angle above horizontal of the observer-terrain line. If the latter angle is larger, there is no intervisibility. Over the duration of a battle with numerous elements this calculation may be made thousands of times. Consequently, not only is the time to prepare the tapes high, but also the time to

compute lines-of-sight once the terrain is input to the model. Considerable effort by developers and users of this model has been expended to streamline this subroutine.

B. A DIFFERENT APPROACH

If a combat simulation is to be used for determining who will win a battle over a specific piece of ground such as the Fulda Gap, then that terrain should be modelled with precision. However, if the model is to be used to determine weapons effectiveness, then representation of a specific terrain is not necessary, and need only be representative of the area to be modelled. Furthermore, as VRI suggested, a single realization (e.g. the Fulda Gap) may produce inaccurate conclusions in a weapons system evaluation. What is needed, therefore, is random terrain representation which can be replicated quickly. However, at the same time it must be representative of a particular "type" of terrain. One approach is to represent hills precisely using a series of mathematical equations. This method would eliminate the inaccuracies of digitized (discrete) terrain but would still have the problem of producing results based on a single realization cf terrain. Moreover, the time involved to model with precision would negate its contribution. At the other end of the terrain modelling spectrum is to use cylinders or cones to represent hills. This is mathematically appealing, but lacks realism even if the method were proven statistically satisfactory. The solution, therefore, lies somewhere in between.

An equation that produces what appears to be a simple hill and at the same time is mathematically tractable is the bivariate normal probability density function. Figure 1 is a sample realization of terrain created and drawn by a computer, using the bivariate normal distribution. The second figure is a contour map of the same terrain, also drawn by the computer.



TERRAIN

Figure 1: Sample of Computer Created Terrain

NEEDELS

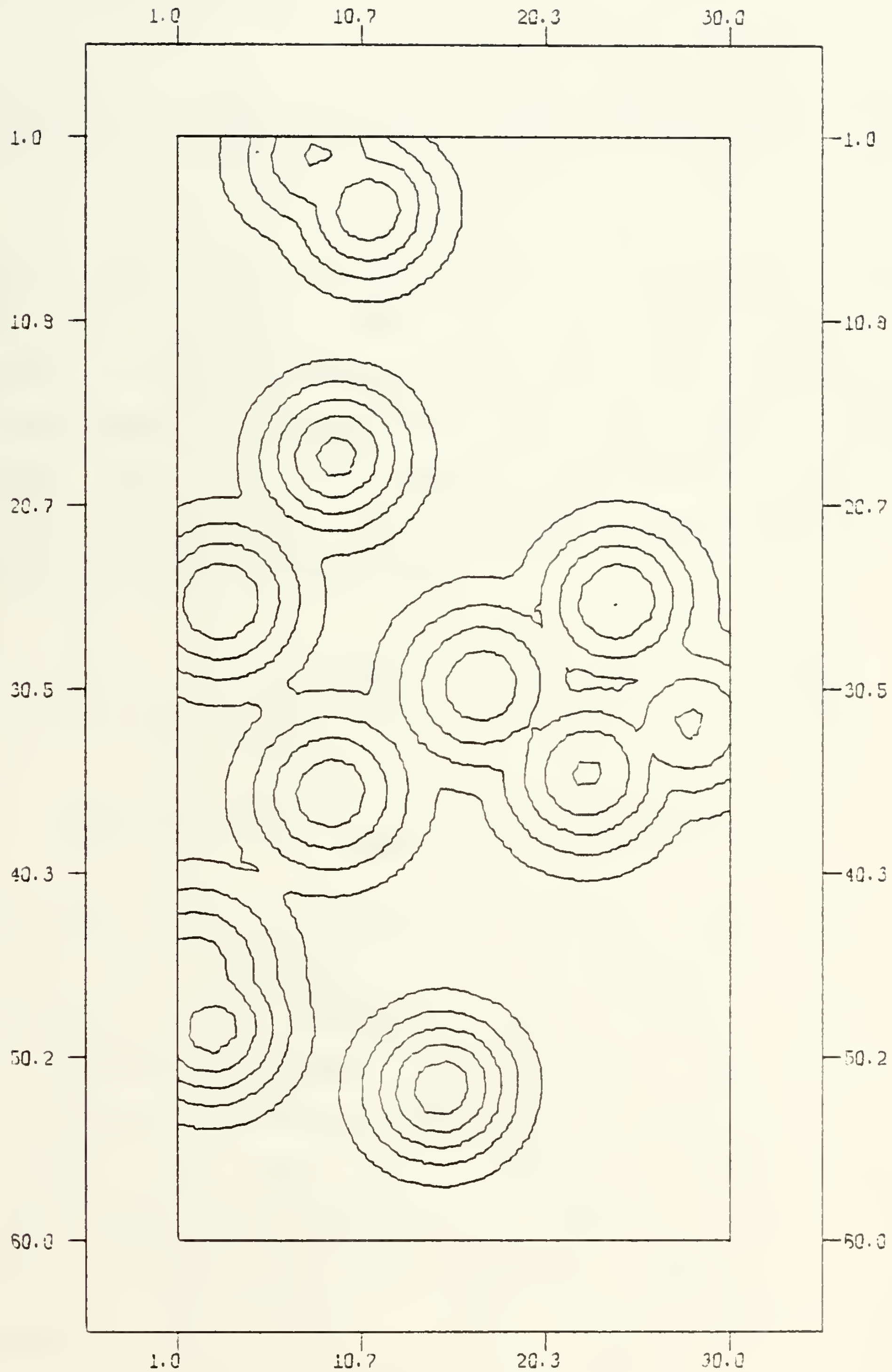


Figure 2: The Sample Depicted as a Contour Map

The common form of the bivariate normal (BVN) density function is as follows:

$$f(X,Y) = \frac{1}{2\Pi\sigma_X\sigma_Y(1-\rho^2)} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{X-\mu_X}{\sigma_X}\right)^2 - 2\rho\frac{(X-\mu_X)(Y-\mu_Y)}{\sigma_X\sigma_Y} + \left(\frac{Y-\mu_Y}{\sigma_Y}\right)^2\right]\right\}$$

Since this equation is that of a probability density function, its integral over $(-\infty,+\infty)$ must equal 1.0. Consequently, as the standard deviations are increased, the hill is flattened. Additionally, even with small standard deviations, the hill will remain less than one unit high. Therefore, to be useful to the modeller, the equation has been modified (MBVN) as follows:

$$(1.2) \quad Z = C \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{X-\mu_X}{\sigma_X}\right)^2 - 2\rho\frac{(X-\mu_X)(Y-\mu_Y)}{\sigma_X\sigma_Y} + \left(\frac{Y-\mu_Y}{\sigma_Y}\right)^2\right]\right\}$$

where C is the peak elevation of the hill.

There are two major advatages of the MBVN equation. First, it is appealing to an observer as a good representation of many different types of hills. Secondly, it has a sufficient number of parameters to vary the shape of a hill without becoming mathematically intractable. The parameters σ_X, σ_Y, and ρ control the shape in the XY plane, the C parameter controls the height of the hills, and μ_X and μ_Y determine the location of the center of the hill on its map. (See Figure 3 for illustrations of the parameters).

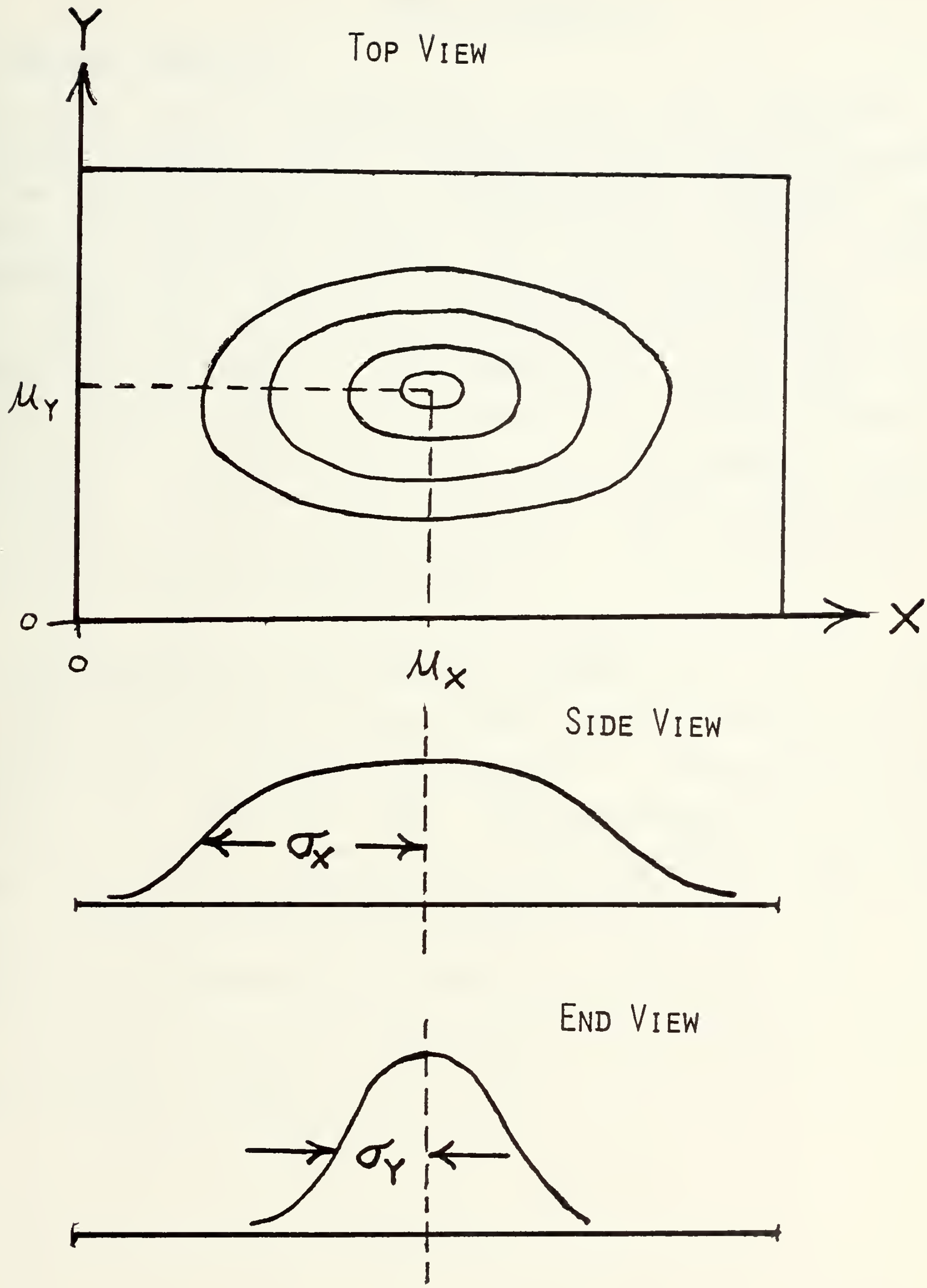


FIGURE 3: SINGLE HILL PARAMETERS

III. MODEL METHODOLGY

A. TERRAIN SIMULATOR

So far in the discussion of the new approach to modelling terrain, only a single hill has been discussed. This has been important as a mathematical foundation, but lacking in practical value. To be of use in a combat simulation, the hills must be created collectively in a size, quantity, and configuration so as to represent a desired type of terrain. For example, a user may want terrain characterized by a few, low rolling hills; or perhaps rugged, mountains with peaks of widely varying elevations. Both can be modelled by adjusting parameters. The remainder of this chapter explains the methodology by examining a working terrain model.

The Simulated Terrain Model (SIMTER) has two parts. The first is the main program which creates terrain and, at the discretion of the user, plots both a three-dimensional drawing and a contour map. Used strictly for terrain generation, it can be used as a preprocessor for other combat models by producing grid points and elevations similar to those provided on computer tapes by WES. The second part of SIMTER is a movement/line-of-sight subroutine (MOVLOS) which moves a target along specified routes across the generated terrain. This part will be discussed in further detail in Chapter V.

In addition to those input parameters used to describe a single hill, others are used to aggregate the hills into a map. These include the following:

1. The dimensions of the battlefield in meters.
2. The grid interval (e.g. 10 meters or 100 meters).
3. The number of hills to be created.
4. How much the peaks of the hills are to vary.
5. How much the spread of the hills are to vary.
6. How many ridge lines, if any, are to be created.

Given the input parameters, the model proceeds as follows.

1. The first step of the program is to randomly select the centers of mass for the desired number of hills. This operation is performed by drawing Uniform (0,1) random numbers and multiplying them by the size of the battlefield in the X and Y directions, thus producing an array of paired grid points. If ridge line are desired, then the random points in either the X or Y direction, but not both, can be biased by drawing random normal deviates about a preselected ridgeline center. Although not a requirement, this particular program makes a hasty plot of the centers of mass points for visual referrence of the random process.

2. The second step is to create the dimensions of each MBVN hill. If the user desires that all the hills be of the same size and shape then this step is complete. However, for most cases the height and spread of each hill will be varied according to an input parameter which is actually the standard deviation for the variations. For example, if a user desires hills which average 100 meters high, but vary about this average value by approximately 30 meters, then 100 becomes the mean value for the peaks of the hills which vary in individual elevations according to normally distributed random numbers whose standard deviation is 30.

The same procedure is done for the spread of the hills in both the X and Y directions.

If only movement and line-of-sight calculations are to be made, then MOVLOS subroutine is called and the program is terminated. A major advantage of this approach to terrain modelling is that no 10 meter, 100 meter, or other grid system is necessary. The terrain representation is continuous; therefore, the elevation at any point on the map can be found without storing any digitized (discrete) map information. Only the hill parameters need be stored.

3. If a matrix of grid values is desirable, as would be the case if the program were used in lieu of the WES computer tapes, then the third part of the SIMTER main program is the creation of an evenly spaced grid system, to include the elevation at each grid line intersection. For all the maps and drawings in this study, a 100 meter grid interval was used.

Since the BVN distribution has some finite value (elevation) in all directions, regardless of how far removed from its center, the elevation of each hill must be computed for each point in the grid system. As each hill is checked for its elevation at any selected point, it is compared with its predecessor. If it is lower, it is discarded; otherwise it is saved for comparison with the next hill's elevation at that same point. After each hill has been searched, only the highest value is stored. It is this value, along with the other maximum values at each grid interval, which make up the terrain surface. If there is a large number of hills, the search process can be streamlined by truncating the MBVN if the height of the density falls below some specified value.

4. The final part of the program is to provide the desired output. This particular program provides both printed and

graphical output. Both the contour map and the three-dimensional drawing were produced by a California Computers Compay CALCOMP Plotter.

B. USER INTERFACE

To be of value to the user, the model must be flexible and its results realistic. In order to demonstrate these capabilities numerous test runs were conducted. The first group of runs used three different average elevations (100, 200, and 300 meters) and three different numbers of hills (5,15,25). The nine resulting terrains are depicted in Appendix A. All maps and drawings represent a 6,000 meter by 3,000 meter battlefield. The second group of runs displays the results of varying the seeds of the Uniform (0,1) random number generator which were used to create the hill centers (Appendis B). The third group (Appendix C) demonstrates the effects of changing the spreads of the hills. For all the tests listed above, the contour interval representing elevation was 50 meters.

IV. LINE-OF-SIGHT

A. THE PRECISE APPROACH

A major advantage of continuous terrain representation is that line-of-sight calculations can be made for any pair of grid coordinates. This is unlike discrete terrain which requires some approximate method such as linear interpolation. If a simple "yes" or "no" answer is desired for the question of intervisibility between any two points on the terrain surface then the mathematics consists of solving the simultaneous equations of a straight line (O-T line) and a curved surface in three space. If a solution exists then there is not line-of-sight, i.e. the line intersects the hills.

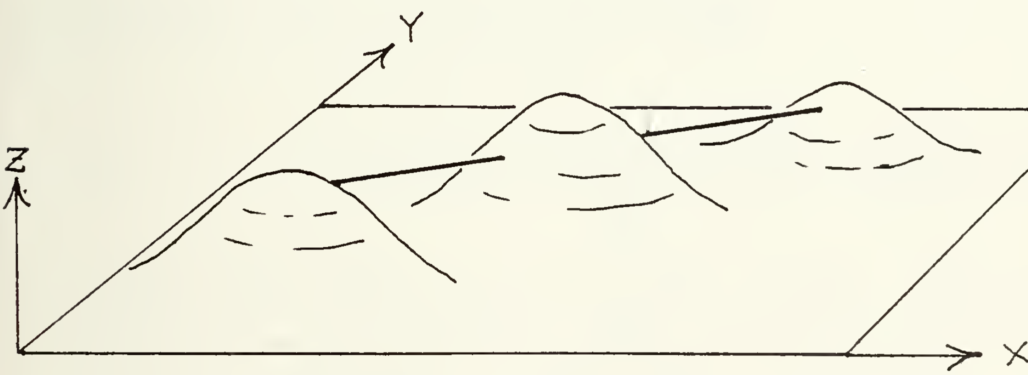


Figure 4: Line-of-Sight

The following is a summary of the procedures.

The O-T line is parameterized as follows:

(4.1) $$X = a_1 + b_2 t$$

(4.2) $$Y = a_2 + b_2 t$$

(4.3) $$Z = a_3 + b_3 t$$

The equations are then substituted into the MBVN equation, resulting in the following expression:

$$a_3+b_3t = C\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{a_1+b_1t-\mu_x}{\sigma_x}\right)^2 -2\rho\frac{(a_1+b_1t-\mu_x)(a_2+b_2-\mu_y)}{\sigma_x\sigma_y}+\left(\frac{a_2+b_2t-\mu_y}{\sigma_y}\right)^2\right]\right\} \tag{4.4}$$

One approach to solving this equation is to take natural logarithms of both sides, thus yielding

$$\ln(a_3+b_3t) = \ln C - \frac{1}{2(1-\rho^2)}\left[\left(\frac{a_1+b_1t-\mu_x}{\sigma_x}\right)^2 -2\rho\frac{(a_1+b_1t-\mu_x)(a_2+b_2t-\mu_y)}{\sigma_x\sigma_y}+\left(\frac{a_2+b_2t-\mu_y}{\sigma_y}\right)^2\right] \tag{4.5}$$

Expanding the left hand side using series expansion of a logarithm amd truncating after three terms produces a quadratic expression. This can be combined with the quadratic on the right and solved using the standard quadratic formula. Unfortunately, the series has very poor convergence with only three terms. Using the average height of a hill as an approximate solution to the series, less than one decimal place accuracy can be expected. The expansion used is provided below:

$$\ln t = \ln a + \frac{(t-a)}{a} - \frac{(t-a)^2}{2a} + \cdots\cdots \quad 0<t\leq 2a \tag{4.6}$$

One approach to improving the accuracy of this solution is to solve the exact solution of a similar 0-T line which is

horizontal and a hill which is circular normal, and use the results to improve the convergence of the series in just three terms. An explanation of this approximation is provided in a later section.

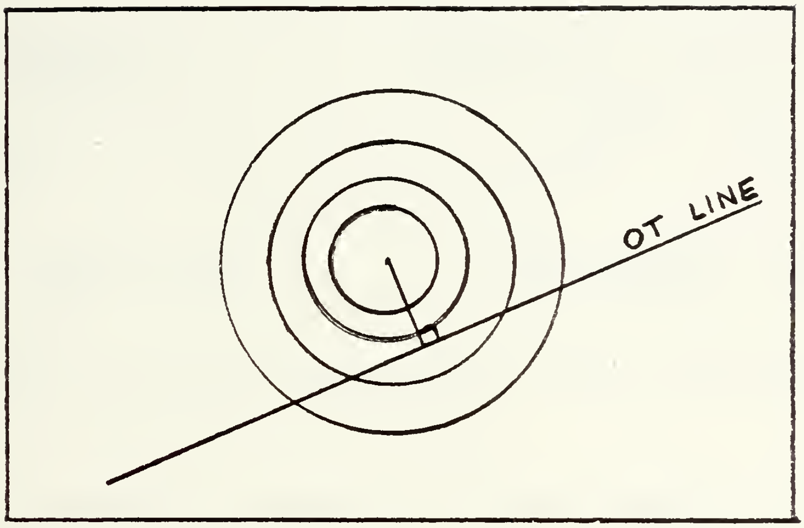
Another way of solving the problem of finding a solution for the parameterized equations of a line and a surface is to use non-linear programming techniques. Using the rapid convergence properties of a Newton search near its solution, a precise answer can be found with minimal iterations.

B. AN APPROXIMATE APPROACH

If intervisibility lines (O-T lines) were horizontal and the MBVN hills were circular, then the mathematics would yield a quick, exact solution. This is done simlpy by projecting the O-T line onto the XY plane and finding the line from the center of the hill perpindicular to the O-T projection (See figure 5). The intersection of these two lines produces a set of coordinates in the XY plane. Finding the elevation of both the O-T line and the hill at this location yields two numbers which can be compared for line-of-sight. If the line is higher than the hill, then intervisibility exists.

Sample calculations to test the accuracy of this approximation yielded two decimal place precision for O-T lines less than 15 degrees from horizontal and MBVN hills whose spreads in the X and Y directions did not exceed ten per cent difference from each other. Fifteen degree angles can be exceeded in two general cases: very high hills and very close ranges. In other words the problem exists when the observer and target are at significantly different altitudes when close to each other. The inaccuracy is illustrated below. The curves represent two-dimensional cross sections of some part of a hill.

Top View



Oblique View

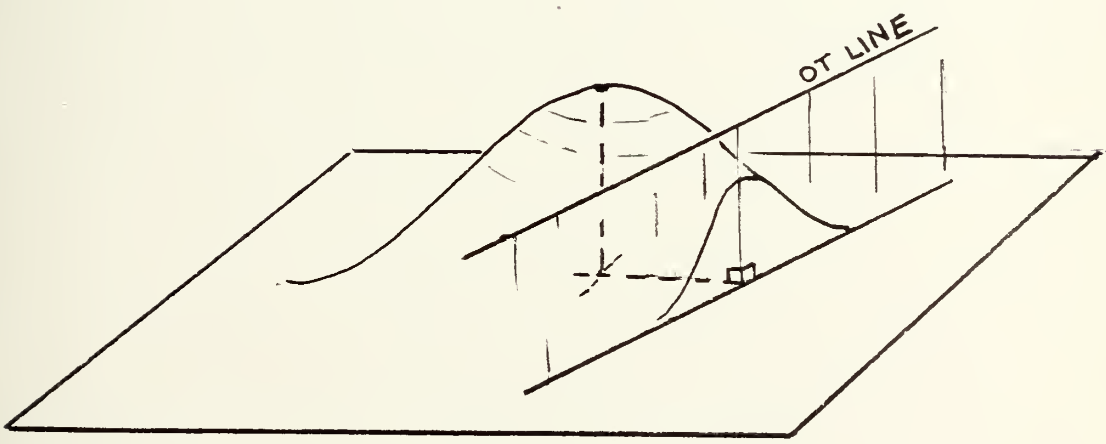


FIGURE 5: Line-of-sight Calculation

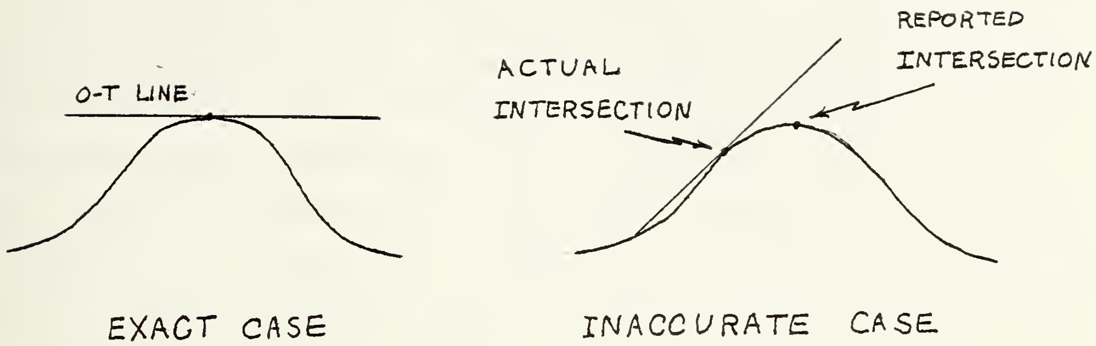


Figure 6: O-T Line Intersection with Hill Cross Section

An efficient solution to the problem is to use the circular normal and horizontal line calculations as long as the situation remains within the confines listed above, and use a Newton search for those few cases which fall outside the desired accuracy.

V. MOVEMENT/LINE-OF-SIGHT SUBROUTINE

A. MODEL METODOLOGY

The movement/Line-of-sight subroutine (MOVLOS) performs three main calculations: movement, line-of-sight, and per cent of target area visible. To perform these functions additional input parameters are required. The first of these is the location of a stationary observer or defender, the second is an array of coordinates which depicts the route of a moving target, and the final input group includes the dimensions of the target. For this program the target is presumed to be rectangular so that height, width, and length are required.

The general mathematics of the line-of-sight calculations were explained in Chapter IV. This section is designed to explain the methodology of applying these mathematics to a combat model. The first step is to determine the elevation of the observer by searching for the highest hill at the observer coordinates. This is accomplished by substituting the observer XY coordinates into the equation of each MBVN hill. The highest resulting functional value is the elevation of the observer. This procedure is likewise carried out for the elevation of the target. Using the three-dimensional coordinates of both the observer and the target, the subroutine forms the equations for the O-T line in three space. The projection of the O-T line onto the XY plane intersects at right angles with lines drawn from the centers of mass of each hill. The coordinates of the intersections are used to determine the elevation of each hill along the O-T line by once again substituting into the MBVN equation of the respective hills.

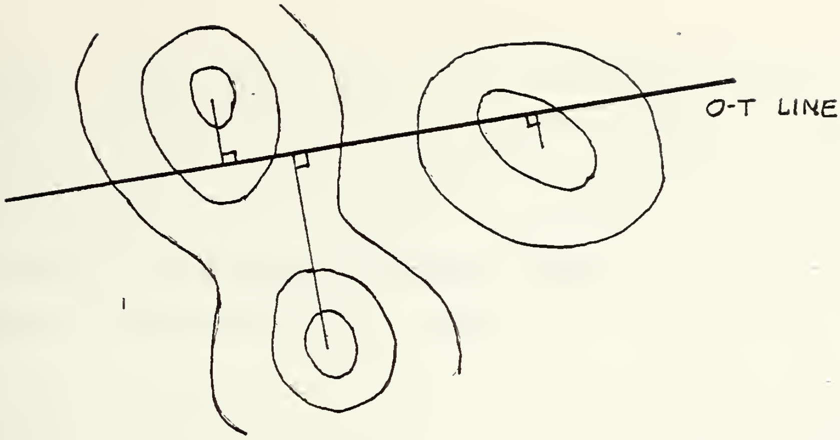
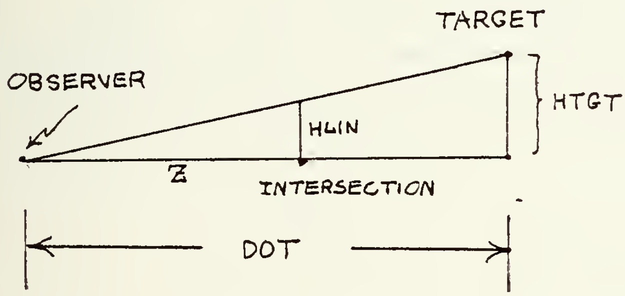


Figure 7; O-T Line Projection

The elevation of the O-T line is also computed at each intersection. This is done using the ratio of sides of similar triangles.



$$HLIN = \frac{(Z)(|HTGT|)}{DOT}$$

Figure 8: Height of O-T Line at Intersection

If any hill is higher than the O-T line along the line's path, then intervisibility does not exist.

To be of use in a high resolution model, the line-of-sight subroutine must answer more than "yes" or "no" to the question of intervisibility. Since the probability of a hit (P_h) is partly a function of target size, the program must determine what effective area of the target is visible to the firer. This, in turn, is a function of the target's dimensions, the per cent of area exposed above the

terrain, and the angle it is facing with respect to the observer. Using basic trigonmetry, MOVLOS computes the area of the target projected in the direction of the O-T line. Two possible target configurations are illustrated in Figure 9. The mirror images of these cases are calculated by using absolute values for the angles.

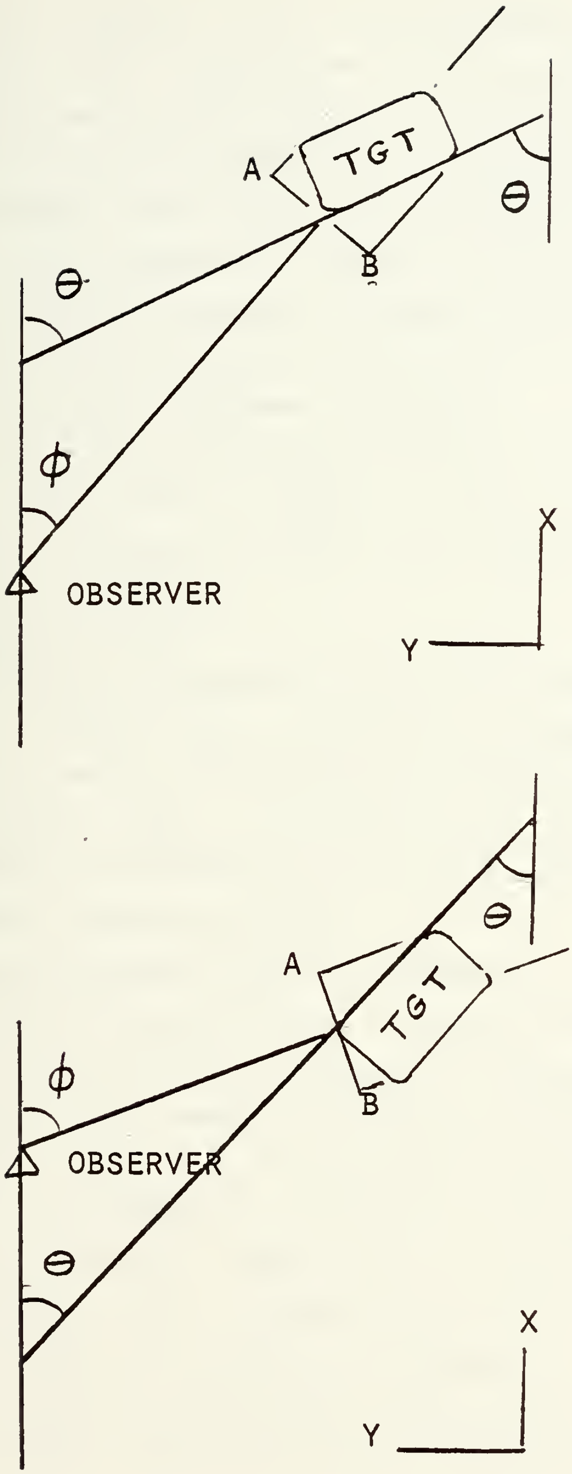
The dynamics of the program are provided by the movement portin of MOVLOS. At discrete time intervals, the target moves along the preselected routes toward its objective. For all runs of SIMTER a one-second time interval was used. At each second an instantaneous velocity and a line-of-sight are computed. If the velocity is not held constant, it is then solely a function of the slope of the terrain. The slope is found by taking the directional derivative (DEL) of the MBVN in the direction of the target. The directional derivative of f(x,y) in the direction is given by

$$\text{DEL} = \frac{\partial f}{\partial X} \cos\theta + \frac{\partial f}{\partial Y} \sin\theta \tag{5.1}$$

For the Modified Bivariate Normal the partial derivatives are as follows:

$$\frac{\partial f}{\partial x} = \frac{C}{(1-\rho^2)}\left[\frac{X-\mu_x}{\sigma_x^2} - \frac{\rho Y-\rho\mu_y}{\sigma_x\sigma_y}\right]\exp\left\{\frac{1}{2(1-\rho^2)}\left[\left(\frac{X-\mu_x}{\sigma_x}\right)^2 - 2\rho\frac{(X-\mu_x)(Y-\mu_y)}{\sigma_x\sigma_y} + \left(\frac{Y-\mu_y}{\sigma_y}\right)^2\right]\right\} \tag{5.2}$$

$$\frac{\partial f}{\partial y} = \frac{C}{(1-\rho^2)}\left[\frac{Y-\mu_y}{\sigma_y^2} - \frac{\rho X-\rho\mu_x}{\sigma_x\sigma_y}\right]\exp\left\{\frac{1}{2(1-\rho^2)}\left[\left(\frac{X-\mu_x}{\sigma_x}\right)^2 - 2\rho\frac{(X-\mu_x)(Y-\mu_y)}{\sigma_x\sigma_y} + \left(\frac{Y-\mu_y}{\sigma_y}\right)^2\right]\right\} \tag{5.3}$$



Target headed left of observer

A-B is projection of target seen by observer

Target headed right of observer

FIGURE 9: Effective Area of Target

The output of the Movement/line-of-sight subroutine includes the elevation, coordinates, speed, per cent exposed, and area exposed at each one-second time increment. Additionally, a summary is provided at the end of the simulation. The information printed is listed below:

1. The number and length of intervisiblity segments.
2. Total distance traveled with intervisibility.
3. Total distance traveled without intervisibility.
4. Total distance traveled.
5. Average distance traveled with intervisibility.
6. Per cent of time in which intervisibility existed.

Refer to Figures 15 and 16 for sample output.

B. A TEST CASE

Although numerous test runs were conducted, a single random sample was initially chosen so that an evaluation could be made of the model's usefulness in a real situation. The intent was to conduct the test in a manner similar to the actual projected use of the model. In this regard, the following steps occurred:

1. The military analyst selected a "type" of terrain.
2. The user applied the terrain description to the model by adjusting the input parameters (Figure 10).
3. The user executed SIMTER without the MOVLOS subroutine in order to obtain a graphical representation of a single random realization of terrain (Figures 11 and 12).
4. The analyst selected a location for the defender (observer), and routes for the attacker. The routes were chosen based on military judgement. For this case routes were based on maximizing cover while attempting to maintain good visual surveillance of the battlefield (Figures 13 and 14).
5. The analyst provided maximum and minimum speeds for the attacker. For this run the minimum speed was two meters per second and the maximum speed was ten meters per second.

These speeds occurred at a 45 degree positive slope and a 45 degree negative slope, respectively. Since the relationship was linear, level terrain velocity was six meters per secon.

6. The user resubmitted the program, this time without creating graphics but including MOVLOS instead. The resulting output (Figures 15 and 16) compared logically with what appeared on the terrain maps.

```
C  SIMTER - TERRAIN SIMULATION
C
C
C  EXPLANATION OF INPUT VARIABLES
C
C *N  NUMBER OF TERRAIN FEATURES (HILLS)
C *XMEL,YMEL  SPREAD OF HILLS (STD DEV FOR BIVARIATE NORMAL)
C *HITE  ELEVATION OF PEAKS OF HILLS
C *RUFX,RUFY,RUFP  IF THE SPREAD AND HEIGHT OF THE HILLS ARE TO BE
C  NORMALLY DISTRIBUTED, THEN THESE PARAMETERS ARE THE STD DEV.
C *RHO  THE CORRELATION COEFFICIENT FOR THE BIVARIATE NORMAL HILLS
C *NOR  DETERMINES IF THE HILLS ARE TO BE CONSTANT IN SPREAD OR TO
C  VARY NORMALLY (0=CONSTANT SPREAD, 1=NORMALLY SPREAD, 2=CIRCULAR
C  NORMAL AND VARYING)
C *NRIDG  DETERMINES NUMBER OF RIDGE LINES, IF ANY,(0,1,2,3)
C *NPEEK  DETERMINES IF THE PEAKS OF THE HILLS SHOULD BE CONSTANT
C  OR VARY NORMALLY (0=CONSTANT, 1=VARYING)
C *INCX,INCY  GRID INCREMENT FOR THE TERRAIN MATRIX
C *XMAP,YMAP  SIZE OF THE GRID SYSTEM (MAP) IN METERS
C *IGRID  DETERMINES IF A GRID SYSTEM IS DESIRED (0=NO, 1=YES)
C *LOS DETERMINES IF THE LINE OF SIGHT ROUTINE IS TO BE CALLED
C *MAPS DETERMINES IF PLOTS ARE TO BE USED
C *M  NUMBER OF TURNING POINTS ALONG THE ROUTE OF THE TARGET
C *RX,RY TURNING POINTS ALONG THE ROUTE OF TARGET
C *OX,OY  LOCATION OF OBSERVER
C *HT,WT,TL  HEIGHTH,WIDTH, AND LENGTH OF THE TARGET
C
```

Explanation of Input Variables

```
21086   451083   701265

 N        MELX       MELY       HITE       RUFX       RUFY       RUFP
12      300.000    300.000    250.000    20.000     20.000     30.000

RHO       NOR       NRIDG      NPEEK      INCX       INCY       XMAP        YMAP
0.0          2          0          1        100        100    6000.000    3000.000
```

Values of Input Variables

Figure 10: Input Data

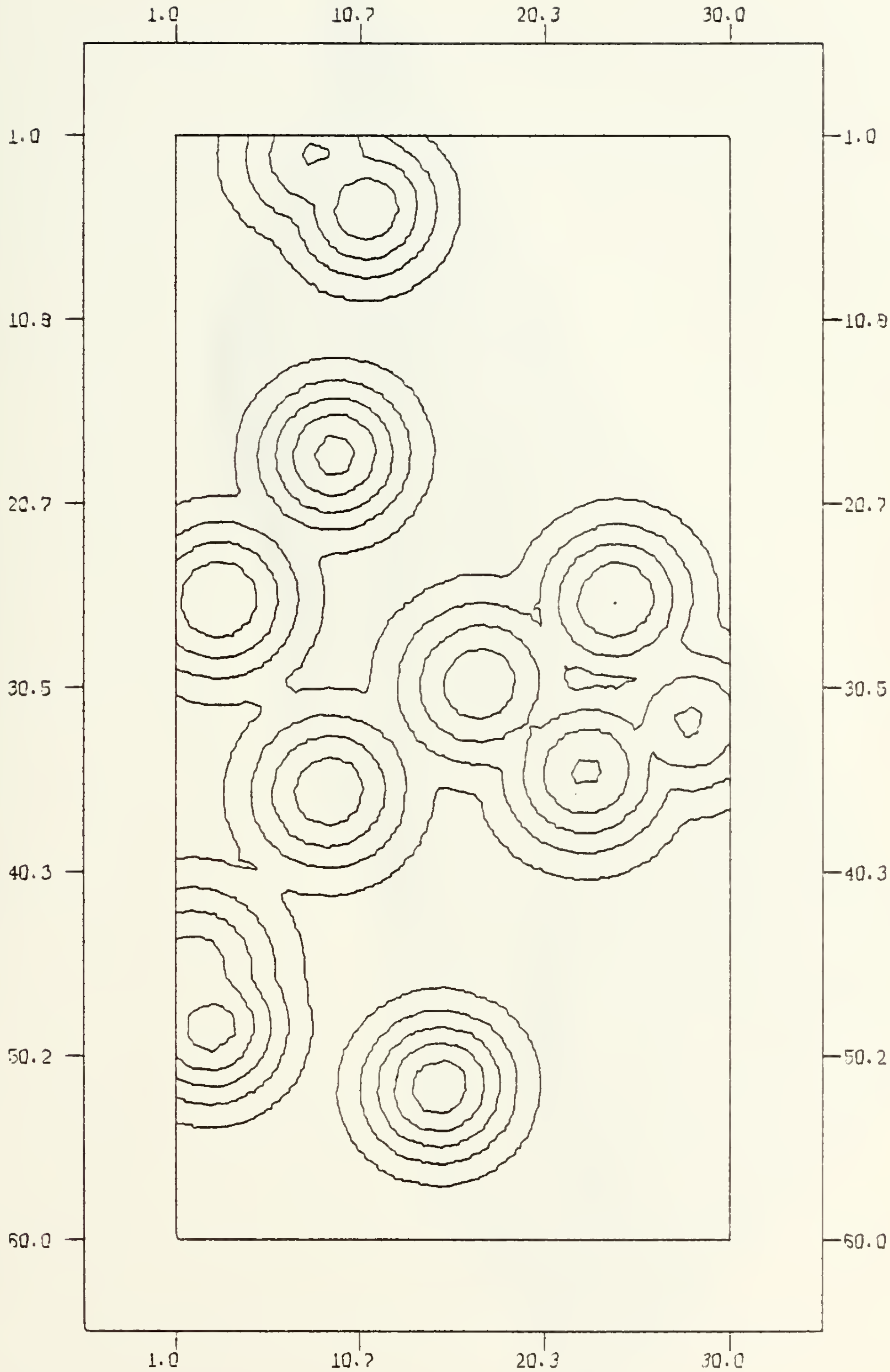


Figure 11: A Sample Terrain

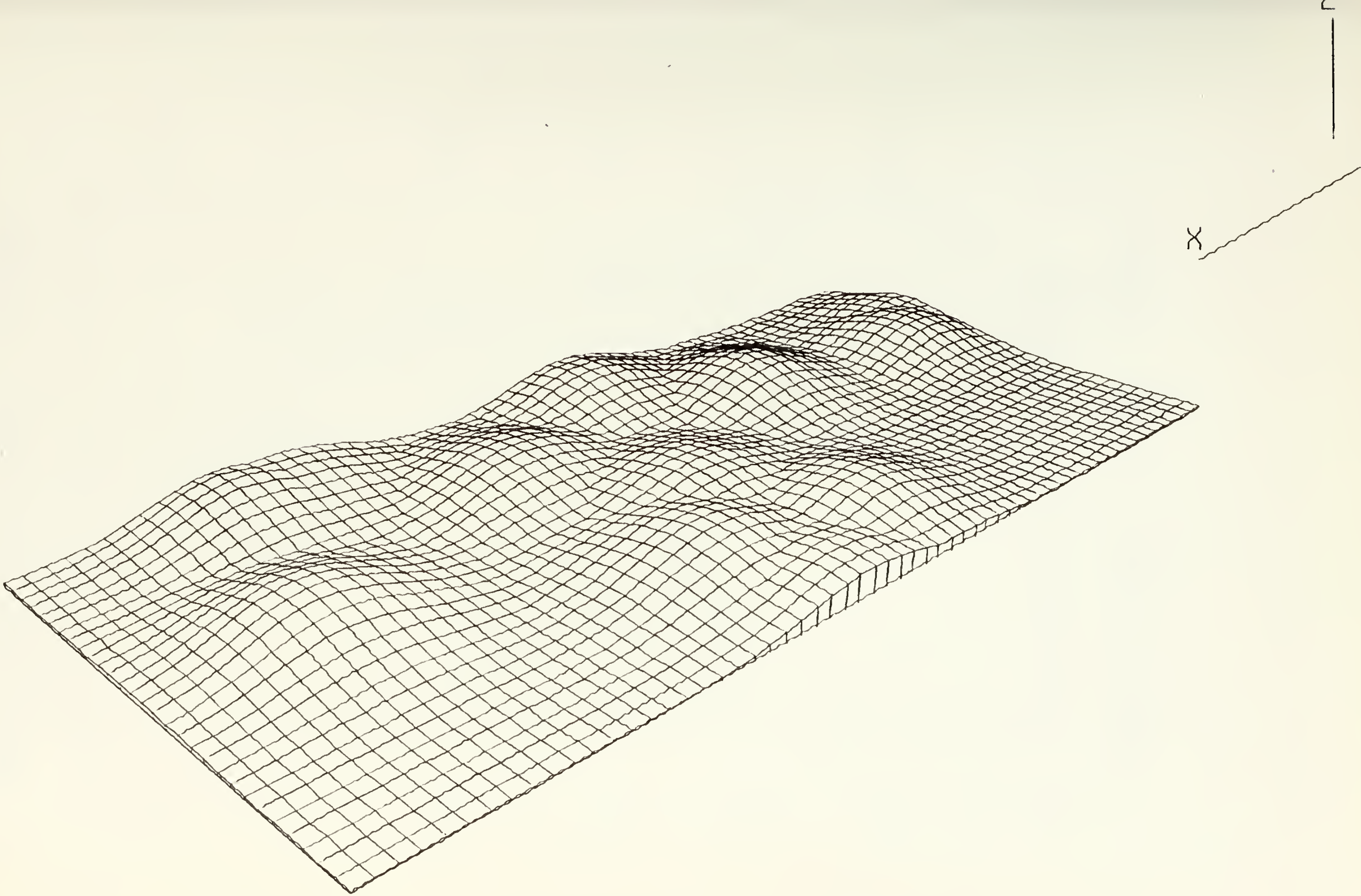


Figure 12: Sample Terrain in Three Dimensions

TERRAIN

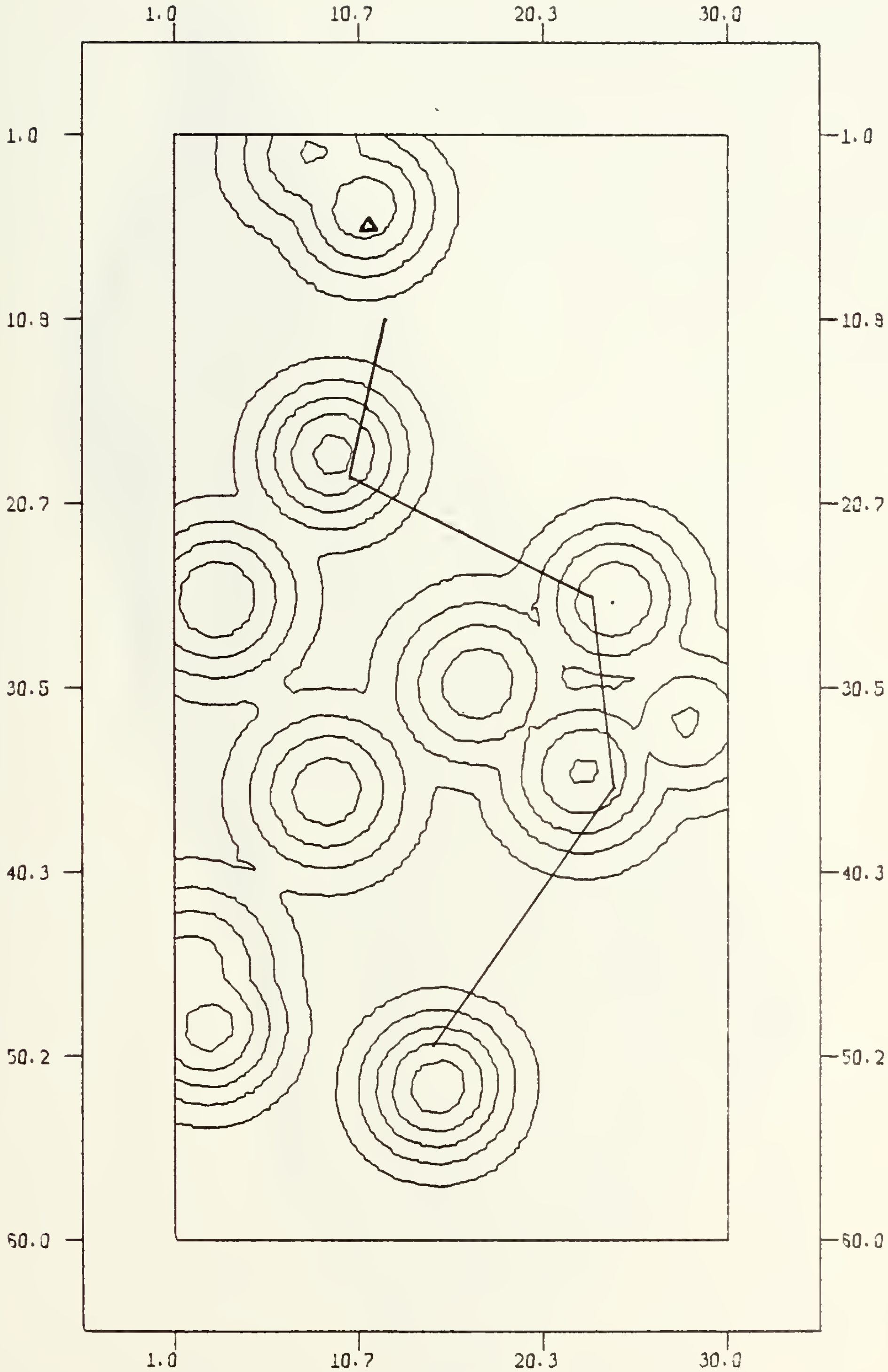


Figure 14: Sample Terrain with Routes

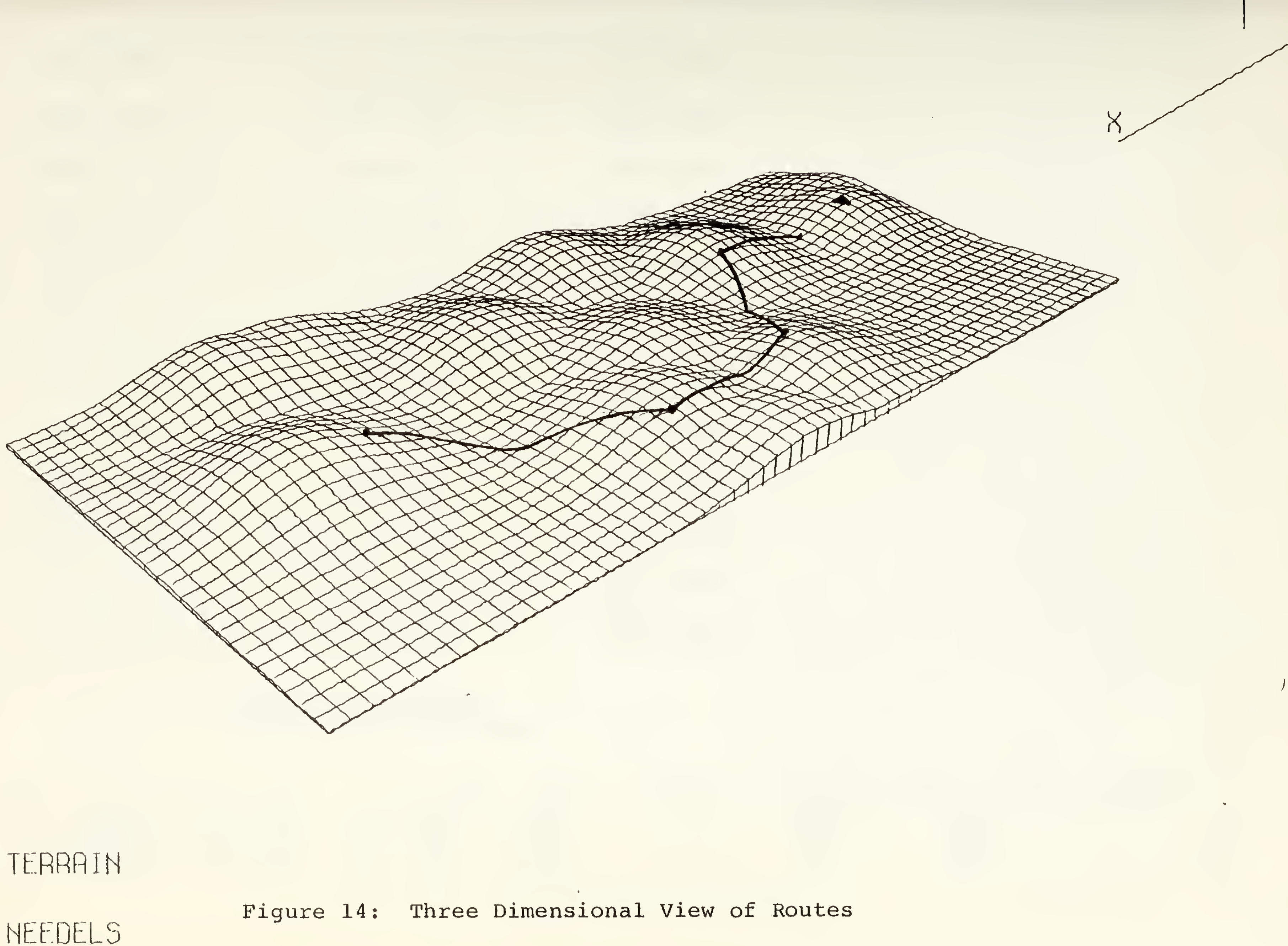


Figure 14: Three Dimensional View of Routes

```
      CENTERS OF MOUNTAINS AND ELEVATIONS AT PEAKS
  2930.15259          1591.02612          237.72595

  5082.01172          1380.26685          287.00928

  3397.61450          2146.65479          260.33130

  1708.57959           832.37500          270.45313

   100.46304           731.26050          207.36163

  2482.80200          2299.93896          251.57468

  4456.73438            79.84763          232.69176

   394.04761           999.23706          242.16846

  4764.23047           181.90198          278.13281

  2482.03125           226.73892          249.65480

  3506.22534           802.86084          238.52051

  3132.94922          2686.08423          208.90152

           OBSERVER COORDINATES
     400.00000  1010.00000   241.92928

TIME    MAP COORDINATES     ELEVATION  SPEED   VISIBLE  % EXPOSED  TARGET AREA EXPOSED
  1   5000.000  1300.000   264.50195   0.0        0        0.0        0.0
  2   4994.699  1303.455   263.92041   6.326      0        0.0        0.0
  3   4989.328  1306.954   263.19897   6.408      0        0.0        0.0
  4   4983.891  1310.499   262.33496   6.491      0        0.0        0.0
```

Figure 15: Output Format and Data

507	2404.755	2260.309	240.98195	6.803	0	0.0	0.0
508	2400.000	2260.000	239.91635	6.872	0	0.0	0.0
509	2397.425	2253.773	238.61424	6.739	0	0.0	0.0
510	2394.823	2247.482	237.18332	6.807	0	0.0	0.0
511	2392.196	2241.129	235.62238	6.875	0	0.0	0.0
512	2389.543	2234.715	233.93068	6.942	1	0.011	0.204
513	2386.865	2228.239	232.10767	7.008	1	0.260	4.879
514	2384.162	2221.702	230.15340	7.073	1	0.530	9.951
515	2381.435	2215.107	228.06822	7.137	1	0.821	15.427
516	2378.683	2208.453	225.85289	7.200	1	1.000	18.803
517	2375.908	2201.743	223.50868	7.261	1	1.000	18.822
518	2373.110	2194.978	221.03719	7.321	1	1.000	18.842
519	2370.290	2188.158	218.44073	7.379	1	1.000	18.862
520	2367.449	2181.287	215.72186	7.435	1	1.000	18.882
521	2364.586	2174.366	212.88373	7.490	1	1.000	18.902
522	2361.704	2167.397	209.92998	7.542	1	1.000	18.922

840	1343.585	1132.553	67.47285	7.550	1	1.000	13.567
841	1336.611	1135.342	64.67104	7.511	1	1.000	13.502
842	1329.673	1138.117	61.95602	7.473	1	1.000	13.437
843	1322.771	1140.878	59.32719	7.434	1	1.000	13.371
844	1315.904	1143.625	56.78409	7.395	1	1.000	13.304
845	1309.074	1146.356	54.32588	7.356	1	1.000	13.236
846	1302.280	1149.074	51.95155	7.317	1	1.000	13.167

INTERVISIBILITY SEGMENT LENGTHS

1	1119.704
2	442.767

DISTANCE UNCOVERED	DISTANCE COVERED	TOTAL DISTANCE	AVG DISTANCE INTERVISIBLE	PER CENT TIME INTERVISIBLE
1562.471	3388.935	4951.406	781.236	0.277

Figure 16: Sample Output Data Continued

C. VERIFICATION

To ensure that the model was responding to all changes in parameters, a series of verification runs were conducted. It was not the intent of these replications to establish the sensitivity of line-of-sight to variations in terrain. The latter was the subject of the previously mentioned VRI study.

The first series of runs consisted of varying the number of hills (5,25, and 25) and the height of the hills (100,200,and 300). Since the same random number generator seeds were used for all nine runs, the 15 hills included the original 5, and the 25 hills included the original 15. This made it possible to use the same routes for all replications without violating tactical doctrine (Appendix D). A matrix of the results is presented in Table 1.

A second group of tests was conducted by varying the maximum and minimum speeds of the attacker over the same range of slopes (-45,+45). The initial (2,10) meters per second was run against (4,16) meters per second. The results yielded no significant difference between the two runs except that the faster target was intervisible 95 out of 501 seconds while the slower was intervisible 160 out of 836 seconds (both 19%).

AVERAGE ELEVATION OF TERRAIN PEAKS

NUMBER OF TERRAIN FEATURES	100				200			
	Route	% Distance Uncovered	Number of Intervisible Segments	Average Distance Intervisible	Route	% Distance Uncovered	Number of Intervisible Segments	Average Distance Intervisible
5	A	86.5	2	2630	A	87.2	2	2678
	B	100.0	1	5370	B	100.0	1	5370
	C	92.6	2	3271	C	92.1	3	2180
15	A	37.8	3	768	A	38.7	3	802
	B	64.8	1	3471	B	41.5	3	741
	C	32.5	2	1138	C	20.2	3	471
25	A	5.9	1	370	A	5.8	1	365
	B	8.2	1	416	B	8.6	1	415
	C	11.5	3	269	C	10.6	3	245

Table 1: Results of Varying Number of Hills and Elevation of Peaks

A third series of runs consisted of varying the spread of the hills in the initial test case, while holding all other parameters constant. As was expected, intervisibility decreased as the hills were widened.

SPREAD	PER CENT INTERVISIBLE
300	27.0
400	19.4
500	7.7

Table 2: Effect of Varying Spread on Intervisibilty

Other runs, such as varying the location (seeds) of the hills while holding routes constant, were executed but not included in this report.

VI. CONCLUSIONS

If exact terrain modelling is not required in a combat simulation, then representative terrain can be created using a modified bivariate normal distribution. Since there is no requirement for survey or photographic interpretation in order to mathematically model terrain, this approach is significantly less costly and time consuming than digitizing terrain. An additional advantage of mathematical representation is that replications of a "type" of terrain can be randomized, thereby improving the statistical level of confidence in a combat model output.

The parameterized terrain is continuous; therefore, the elevation at any location is exact and not a linear interpolation between discrete points. This makes it possible for line-of-sight calculations to be more accurate. On the other hand if the program is to be used as a terrain preprocessor for a high resolution combat model, then SIMTER can produce a digitized output from the parameterized representation.

The results of the tests indicate that the parametric representation of terrain is both useful and realistic. In view of these conclusions the following action is recommended:

1. Parameterized terrain should be run against digitized actual terrain in a high resolution model such as DYNTACS or AMC 74. Sufficient replications of the randomized parametric terrain should be conducted in order to establish steady state results.
2. Distributions other than the bivariate normal should be examined, e.g. the beta.
3. The feasibility of representing actual terrain with the MBVN should be examined by preselecting centers of hills as they appear on a map.

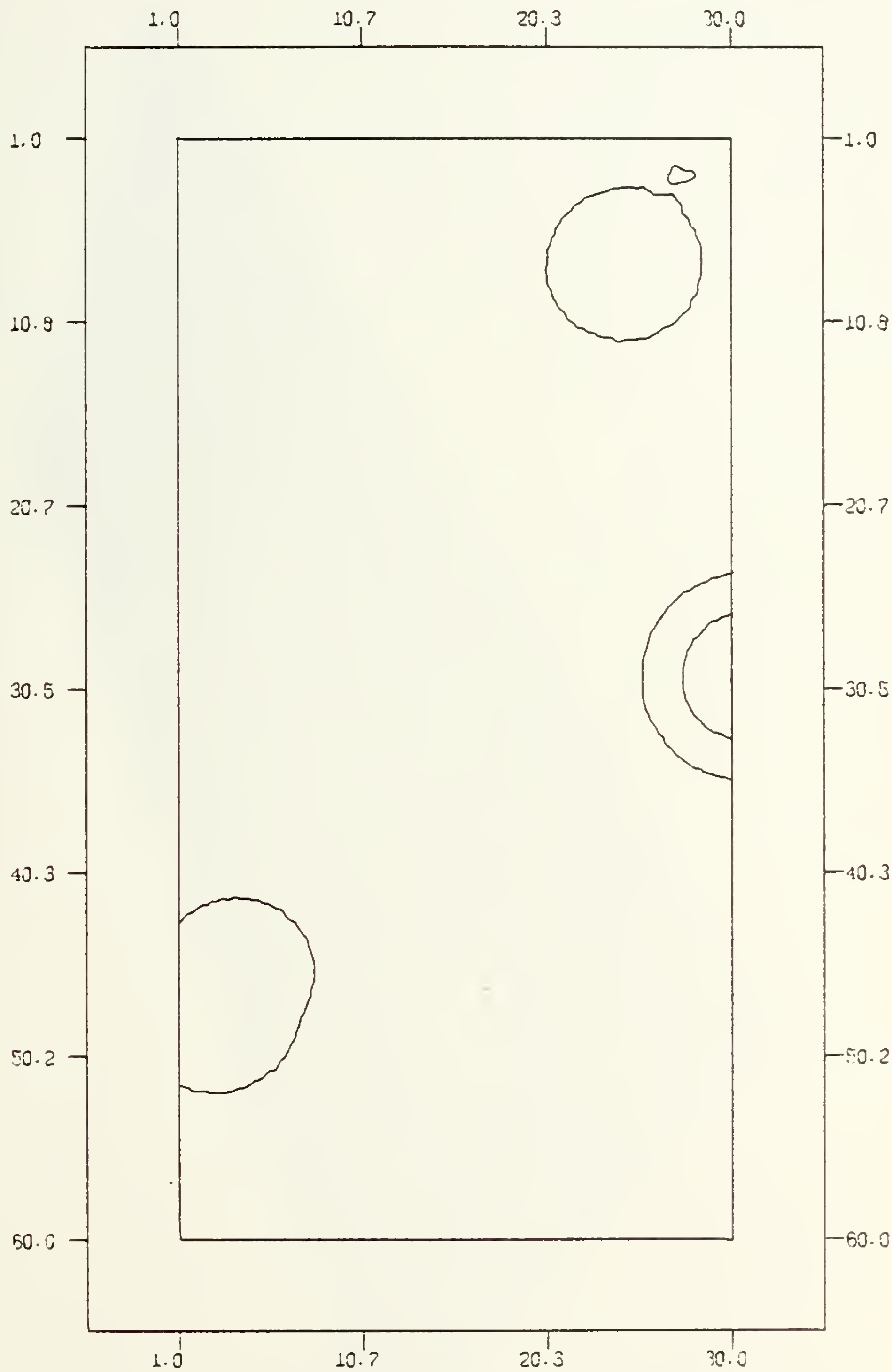
4. The SIMTER simulation should be evaluated for use as a mobility model. Routes can be selected or readily changed commensurate with vehicle performance.

APPENDIX A

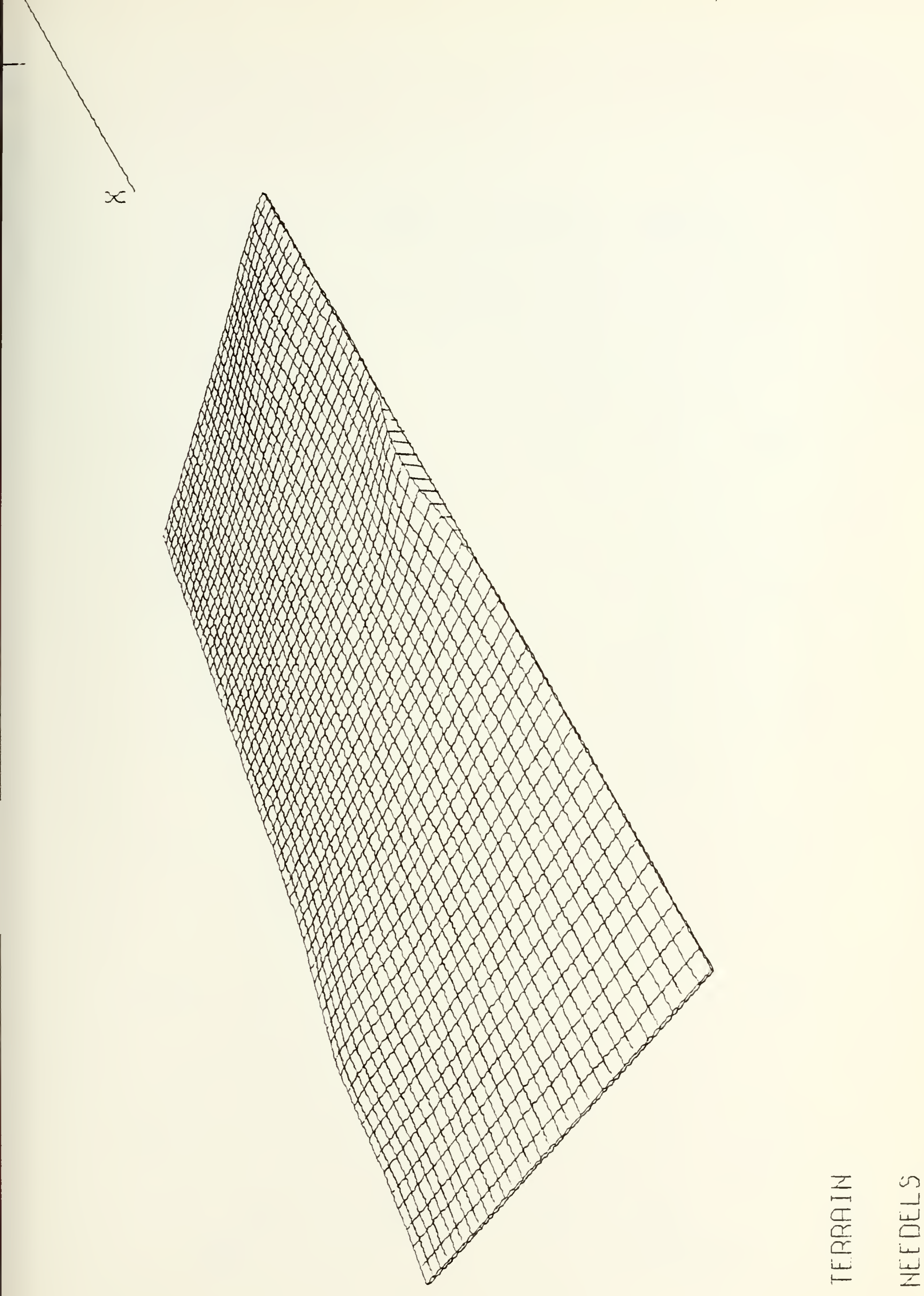
MATRIX OF INPUT VARIABLES

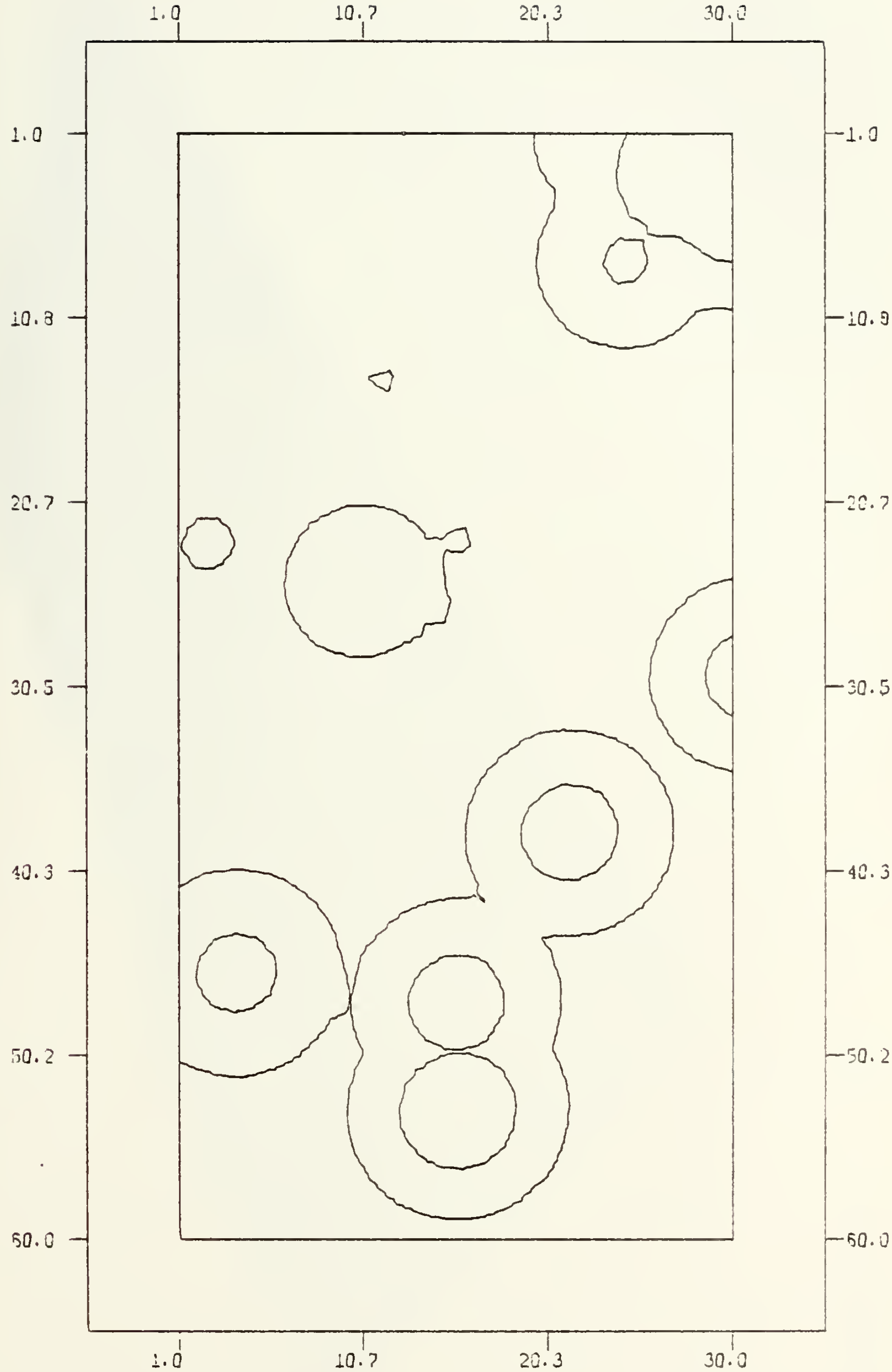
The graphs included in this appendix illustrate variations in number of terrain features (N) and average height of hills (PEAK). The graphs are presented in pairs; the first being the contour map and the second being its three dimensional representation. The order of variation is listed below.

N	PEAK
5	100
15	100
25	100
5	200
15	200
25	200
5	300
15	300
25	300

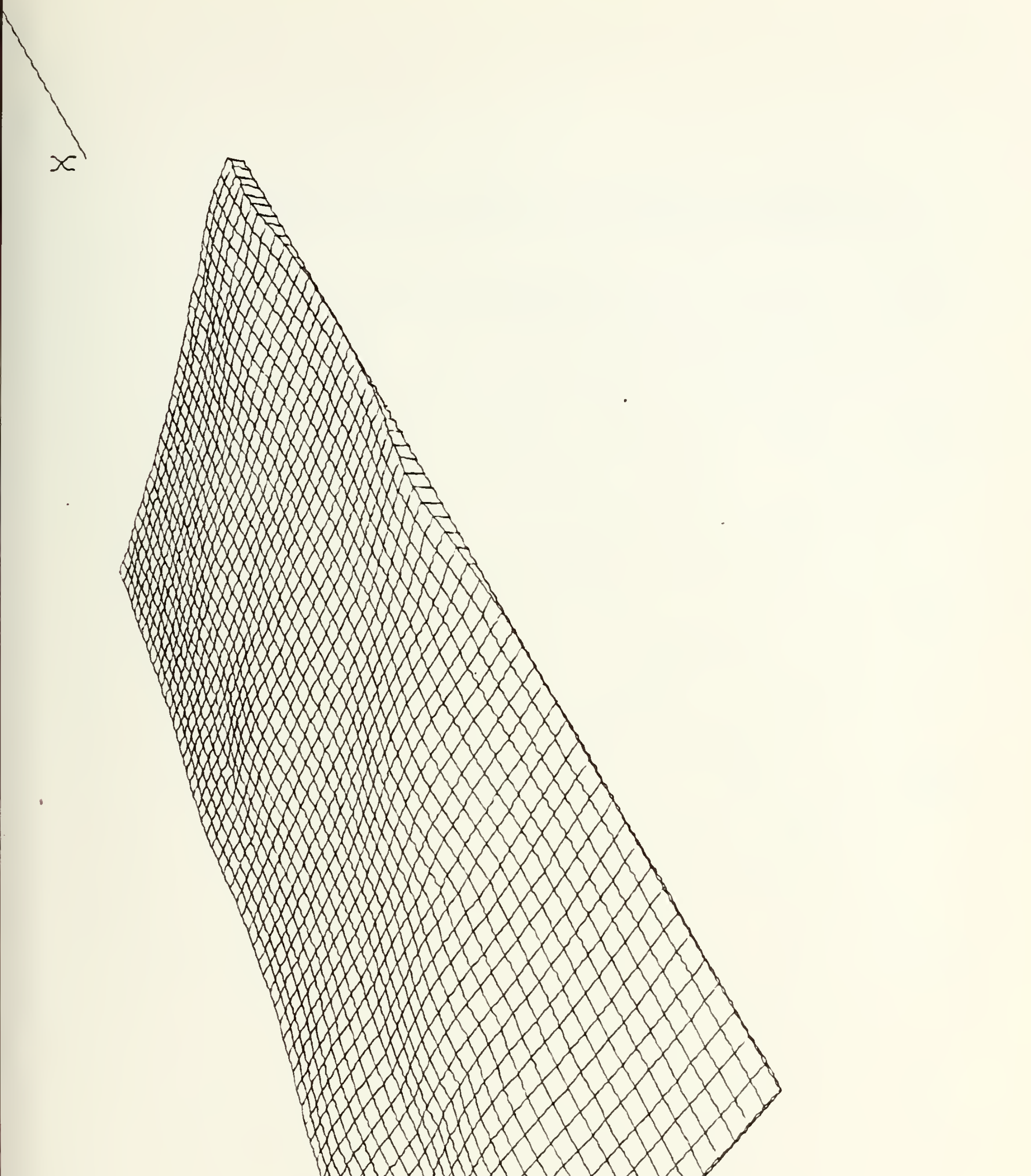


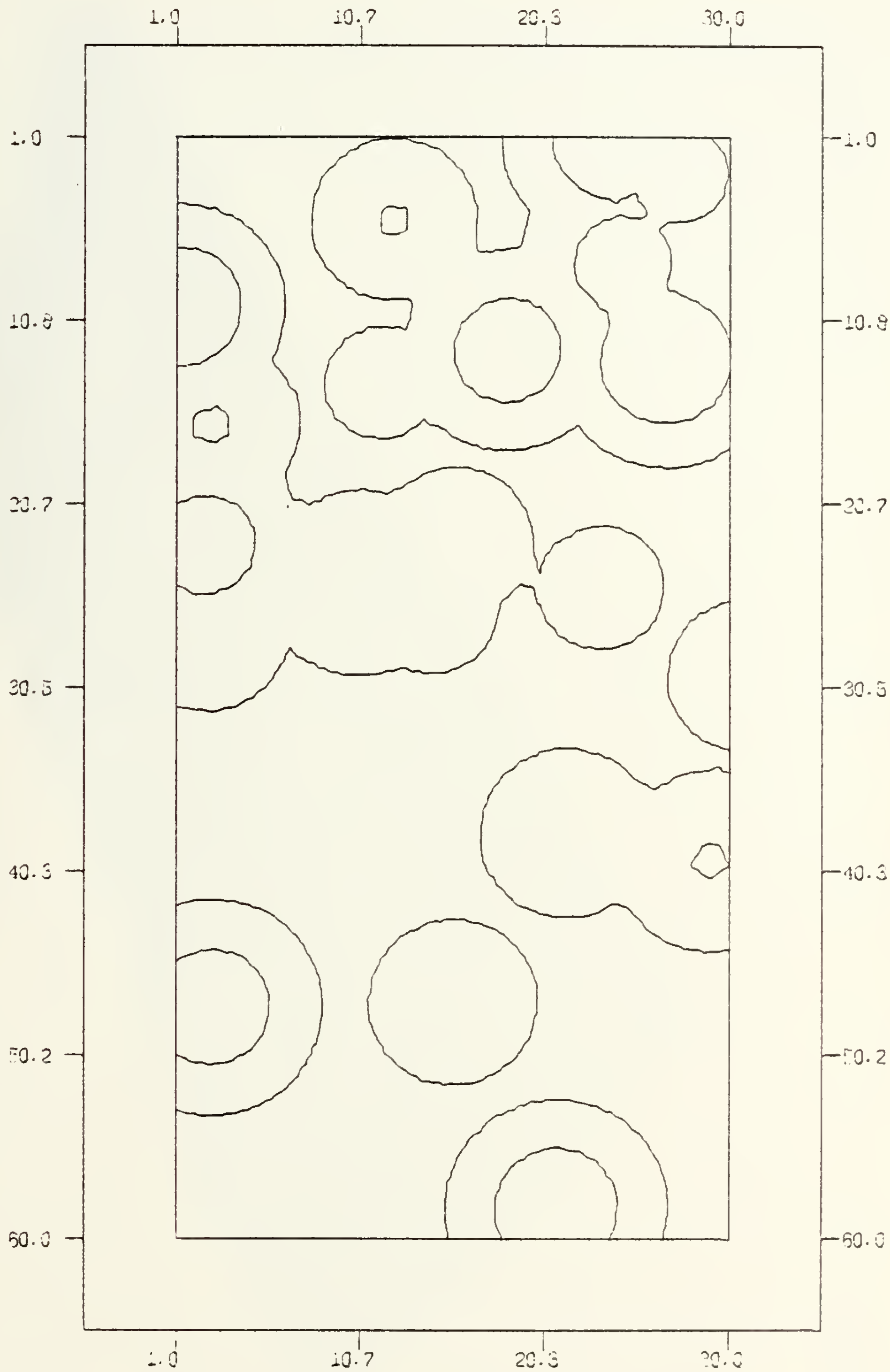
N=5 PEAK=100


x
TERRAIN
NEEDELS



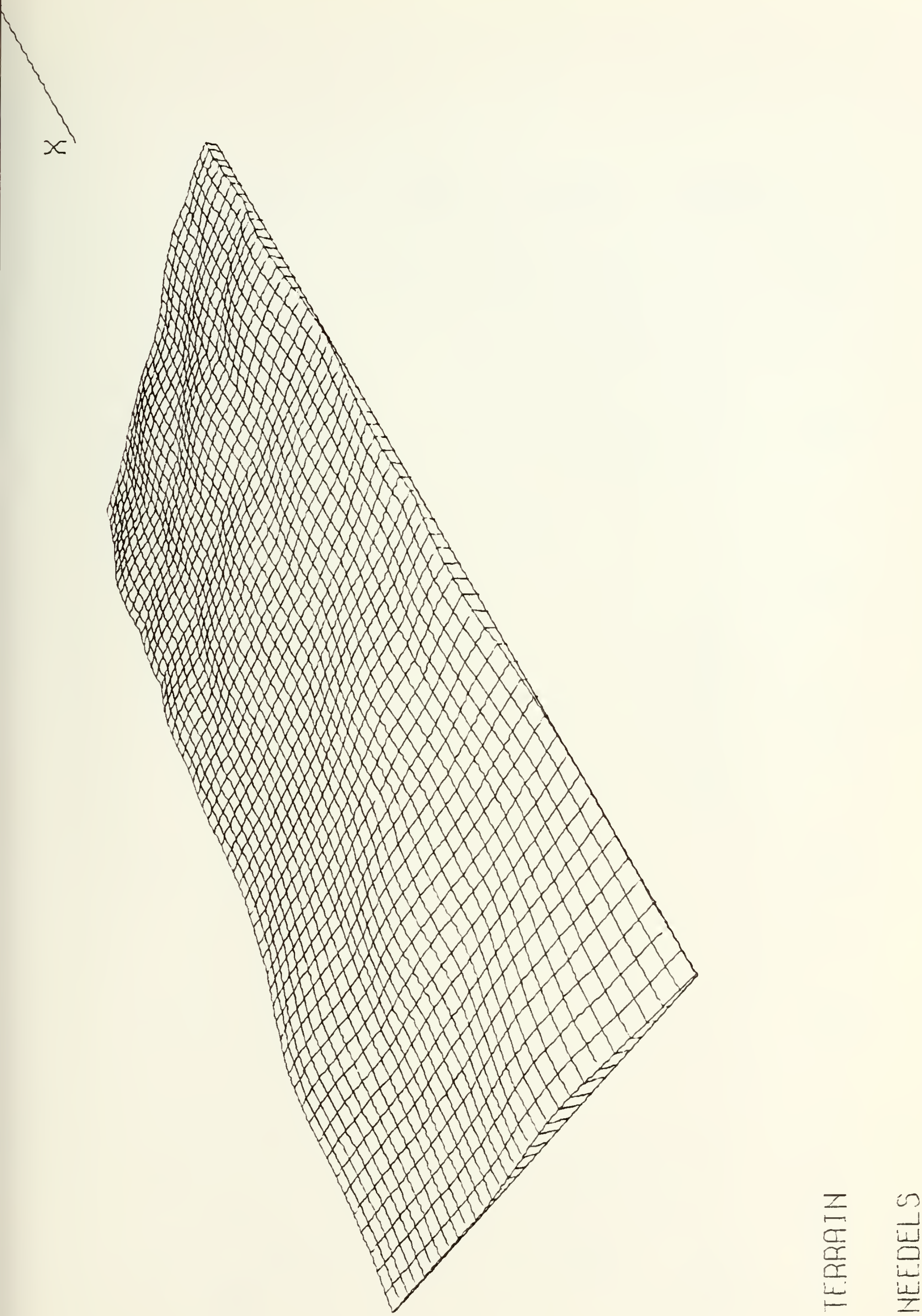
N=15 PEAK=100

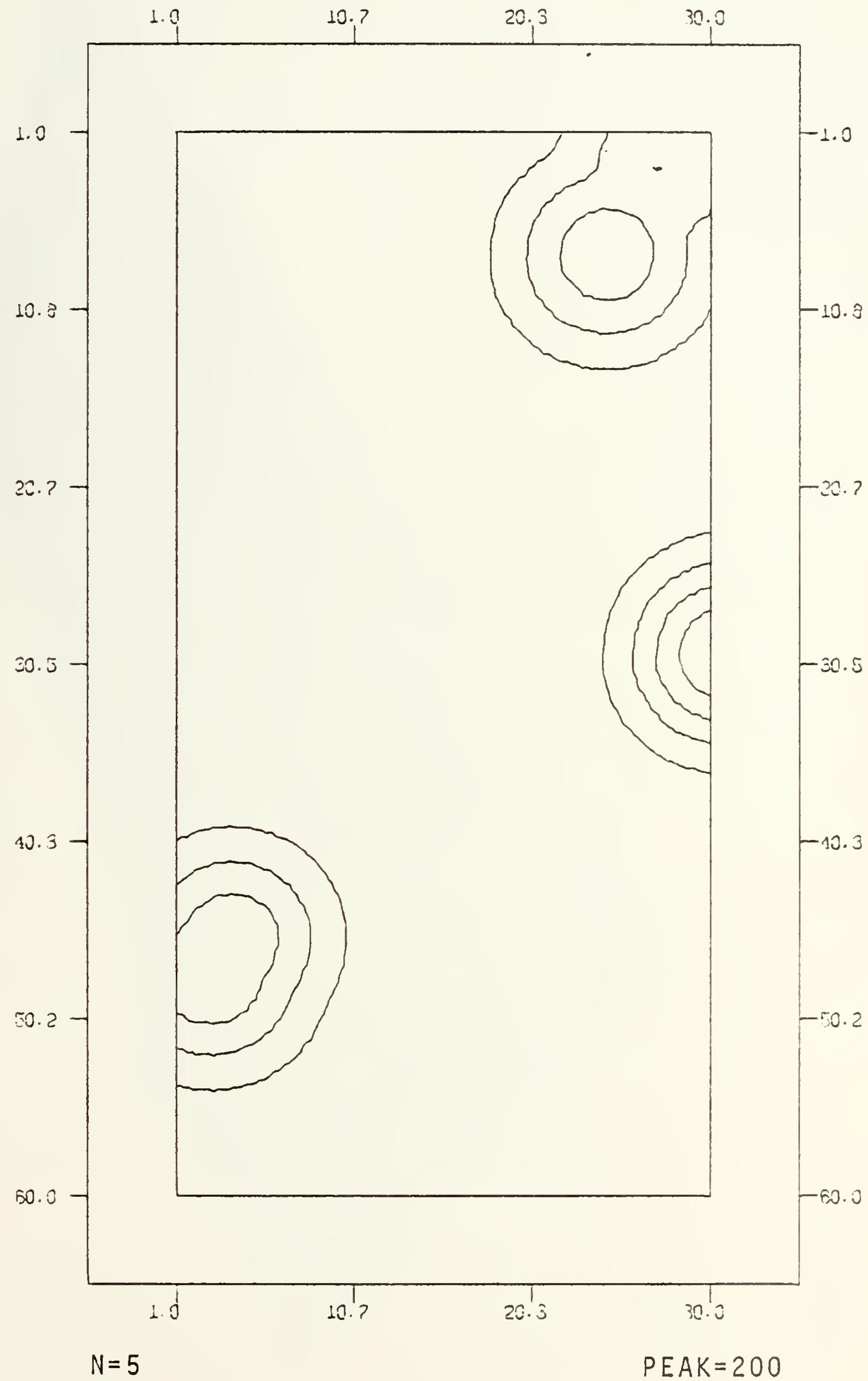

x
TERRAIN
NEEDELS

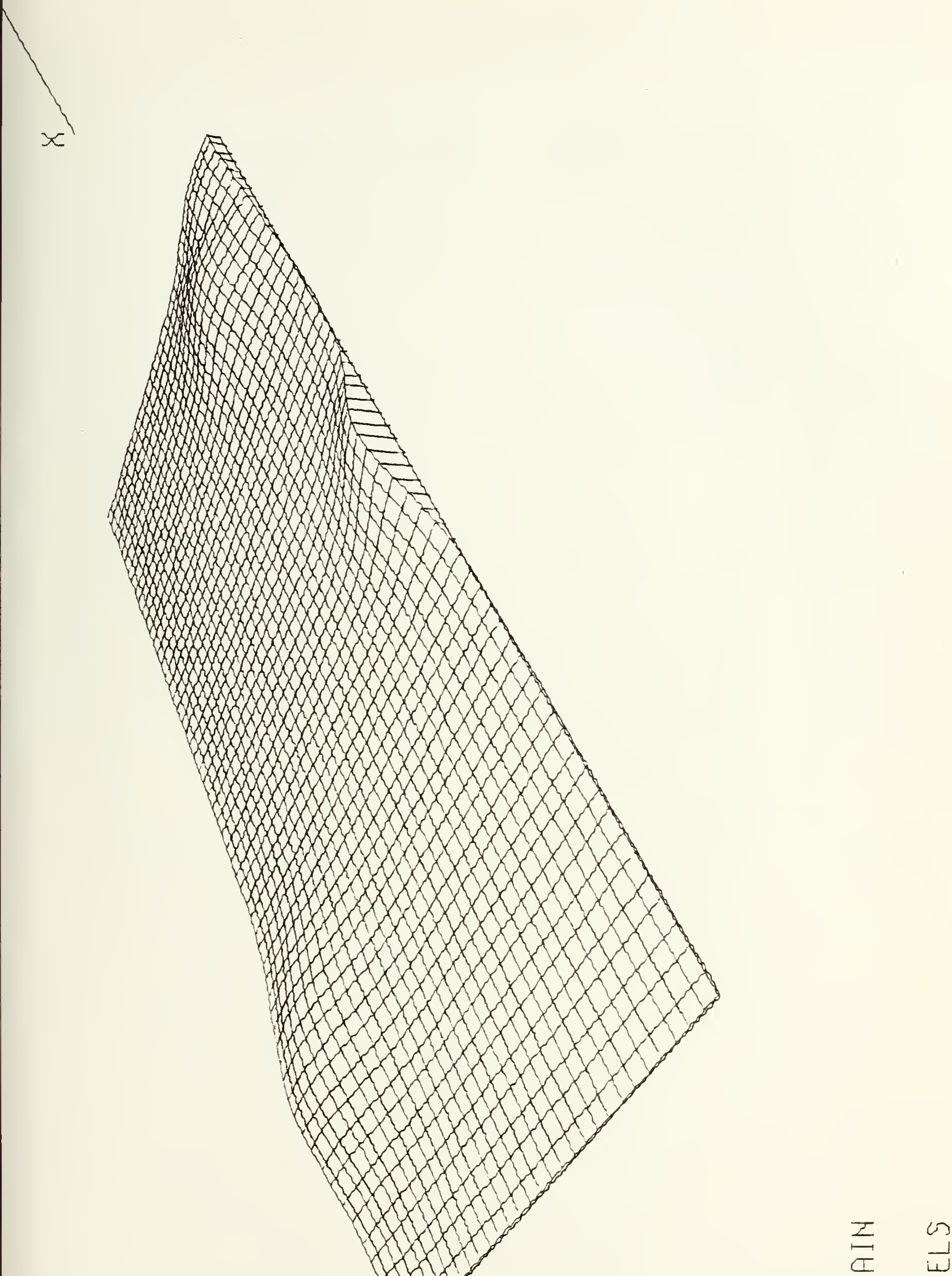


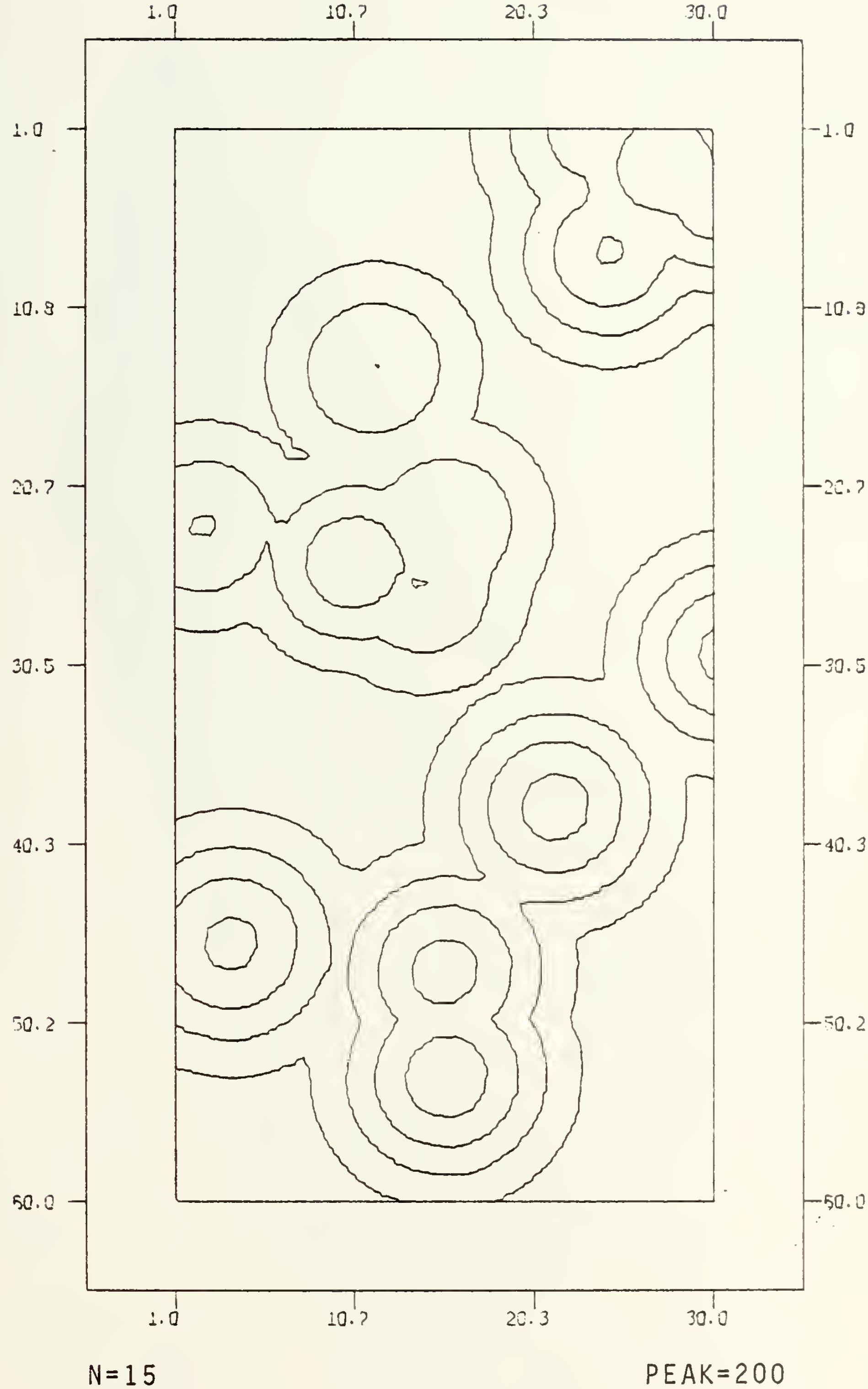
N=25

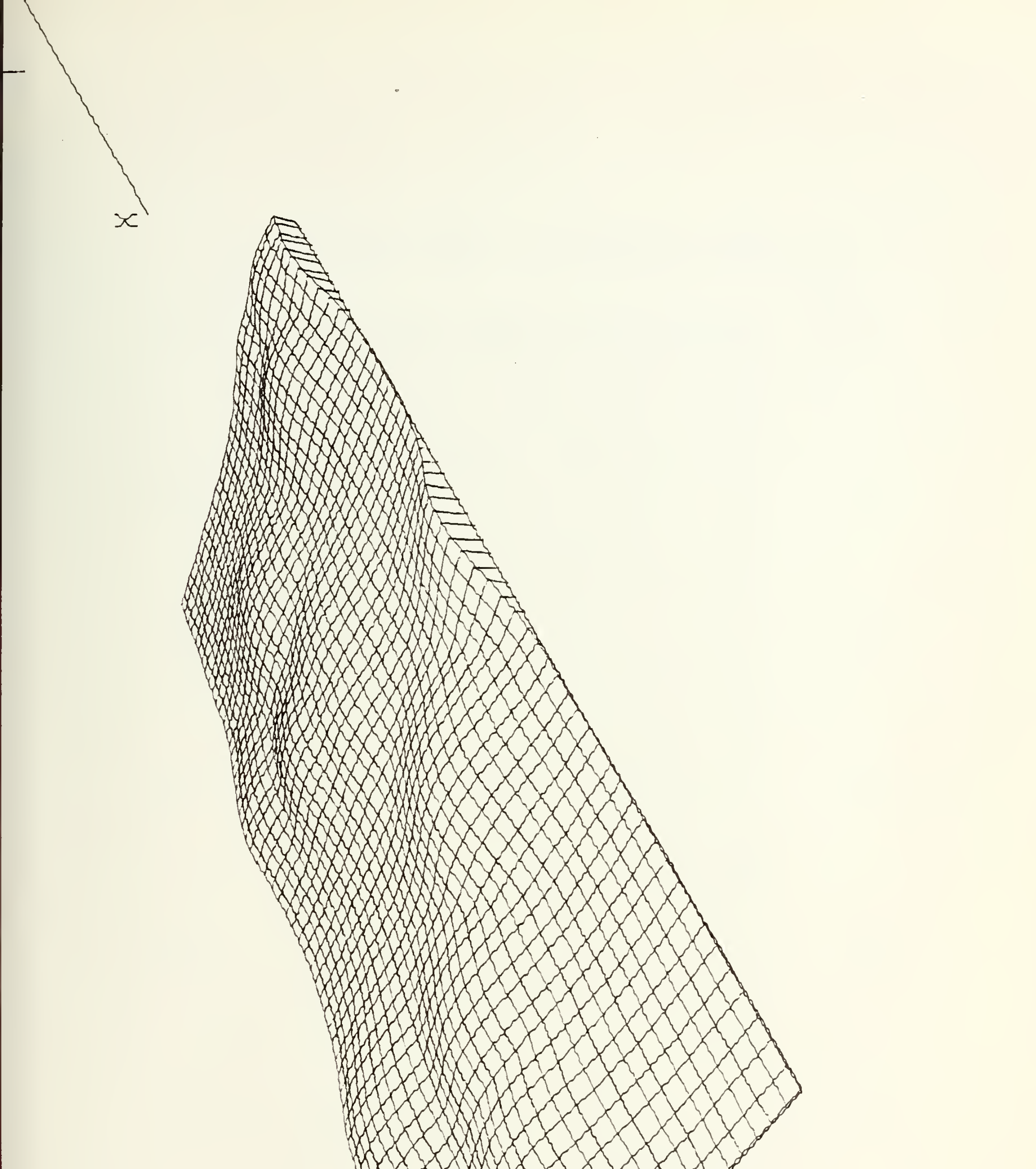
PEAK=100

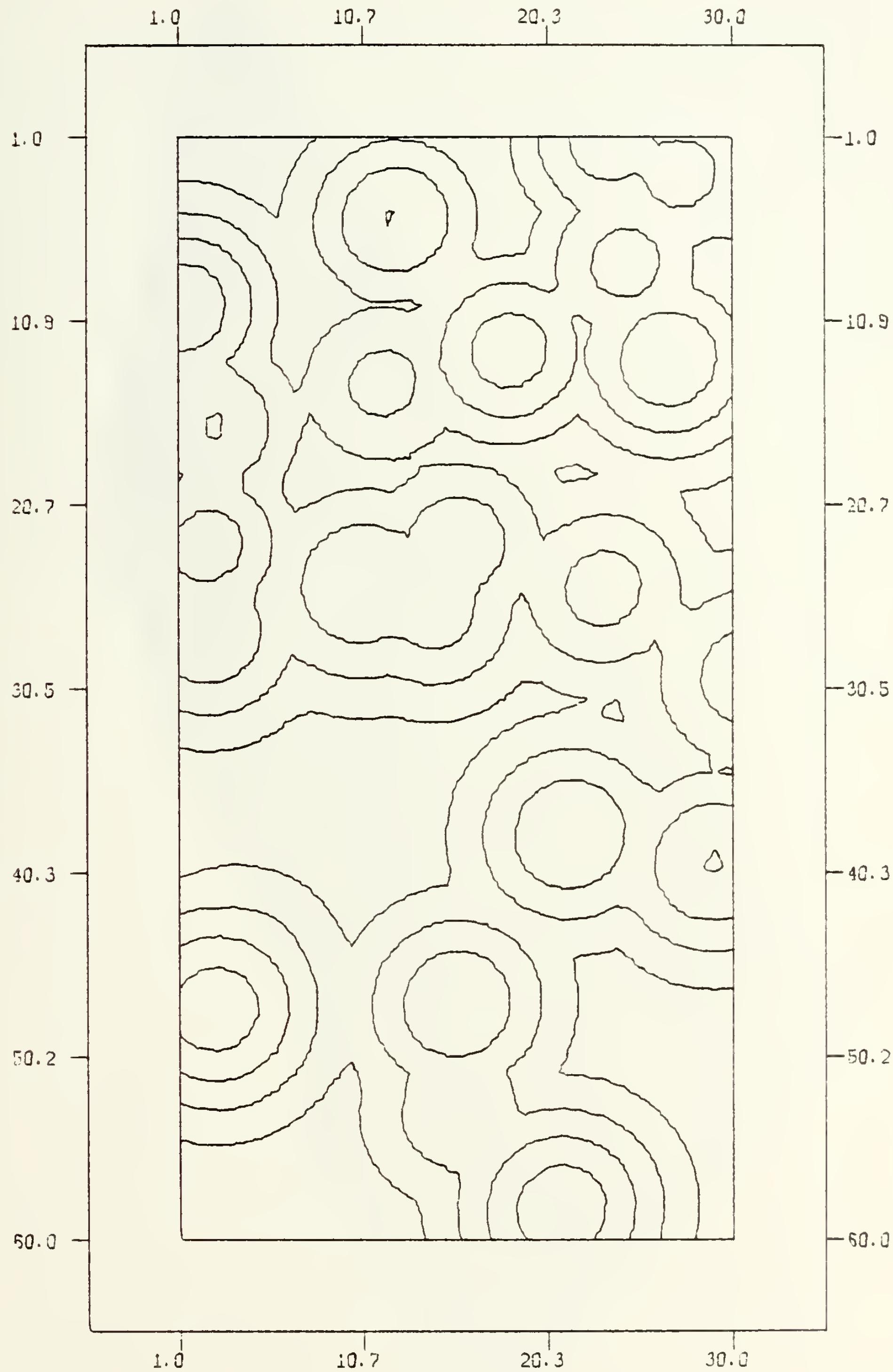

X
TERRAIN
NEEDELS


1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0
N=5
PEAK=200


x
TERRAIN
NEEDELS

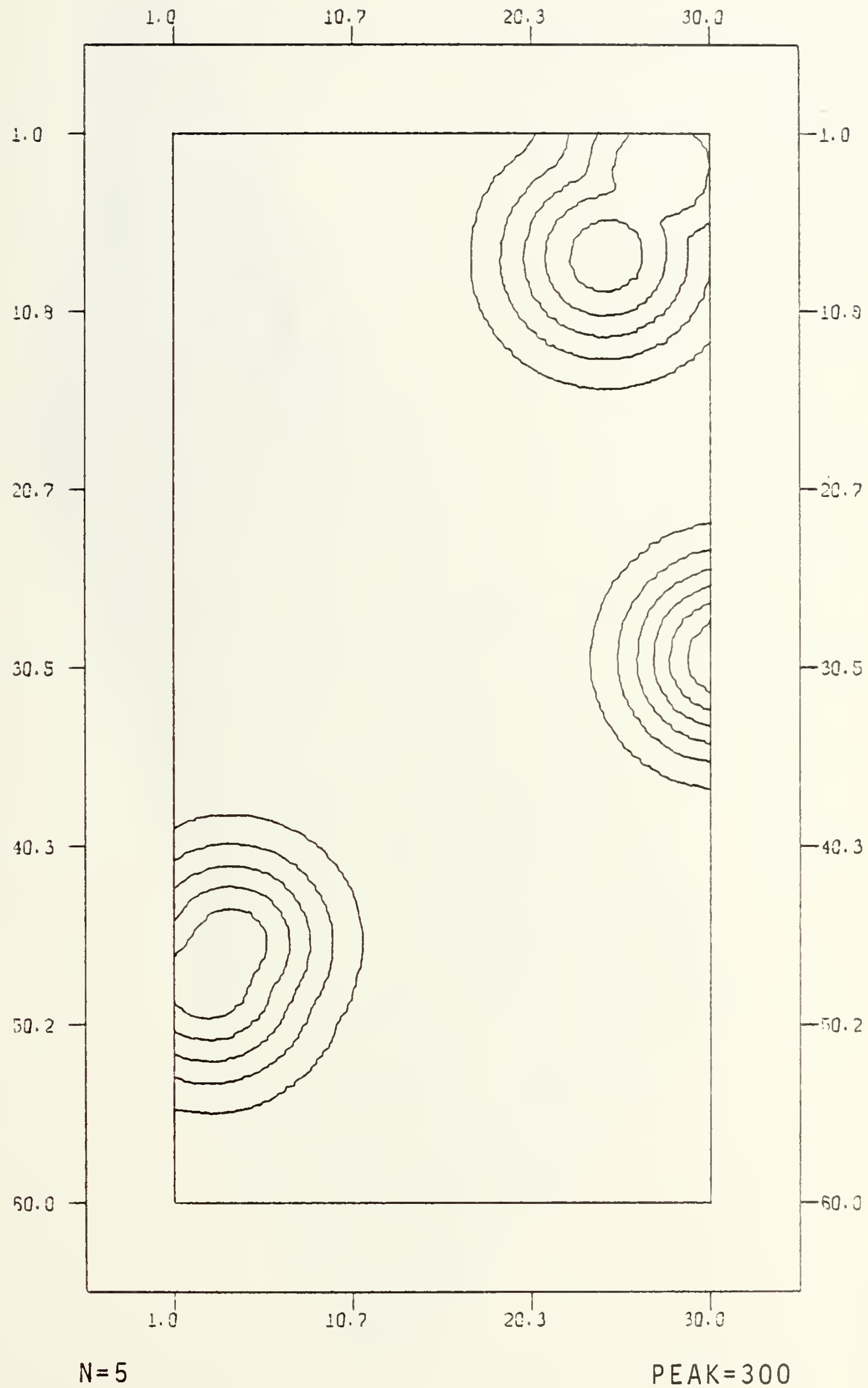

1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0
N=15
PEAK=200

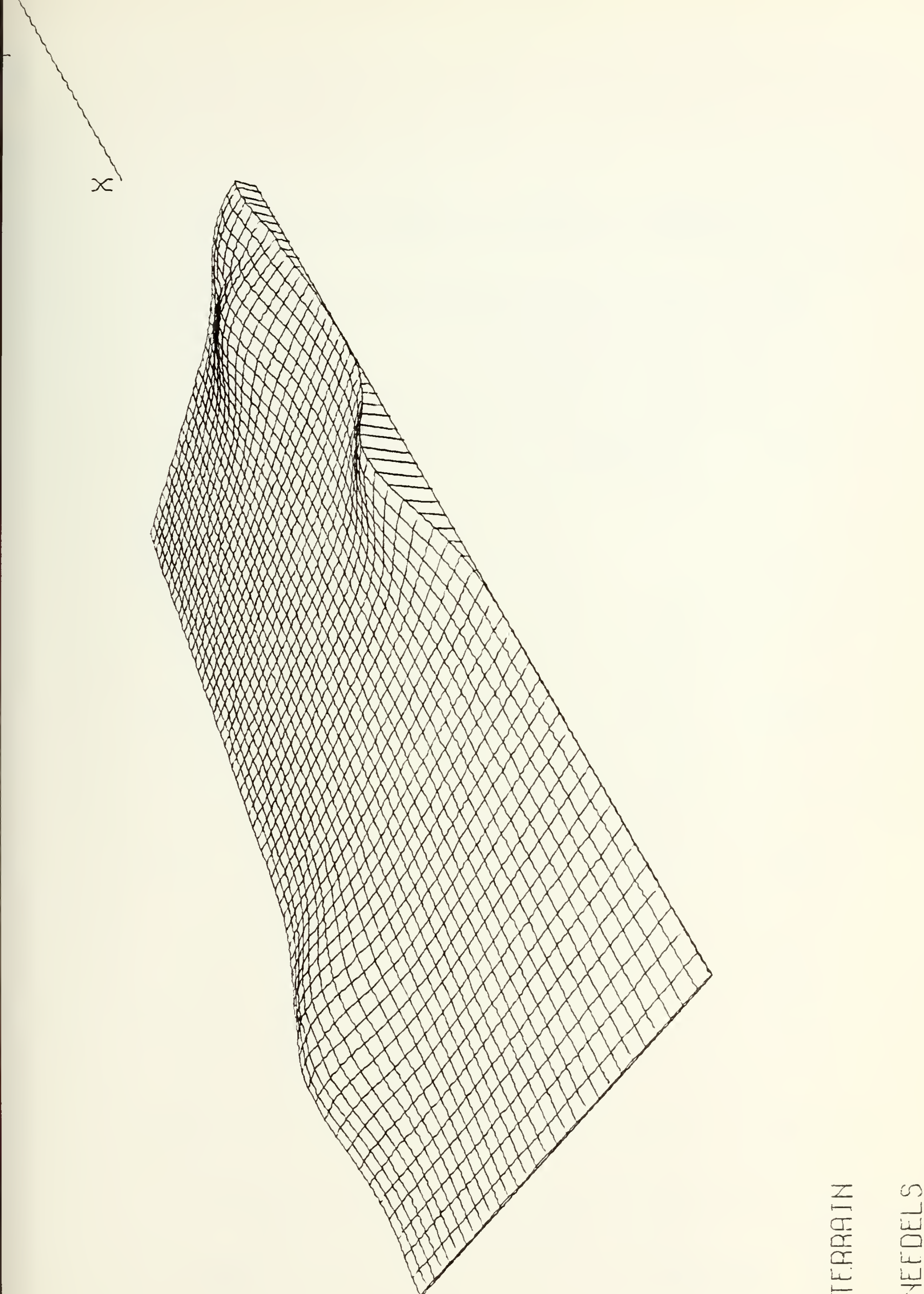

x
TERRAIN
NEEDELS

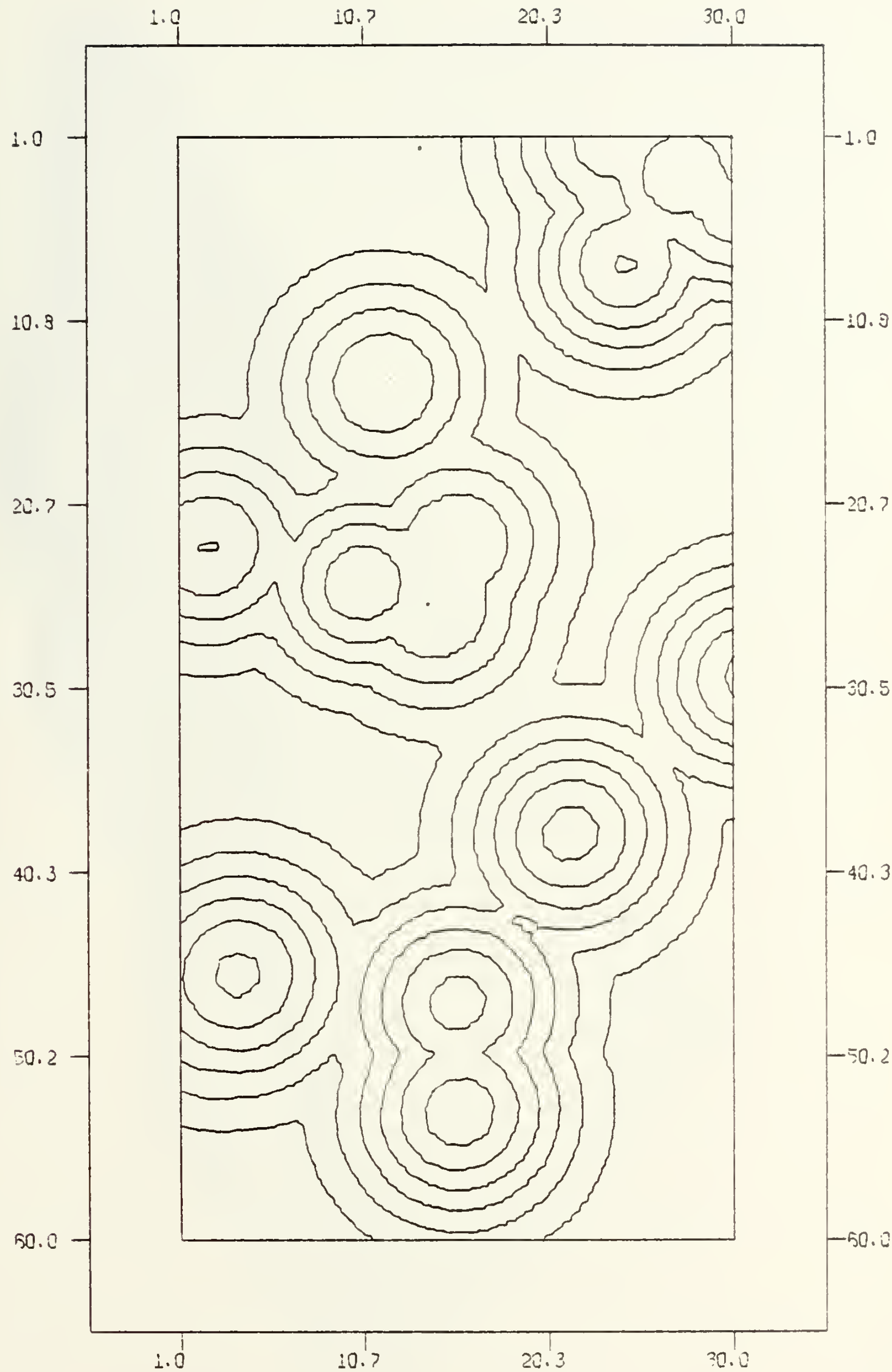


N=25

PEAK=200

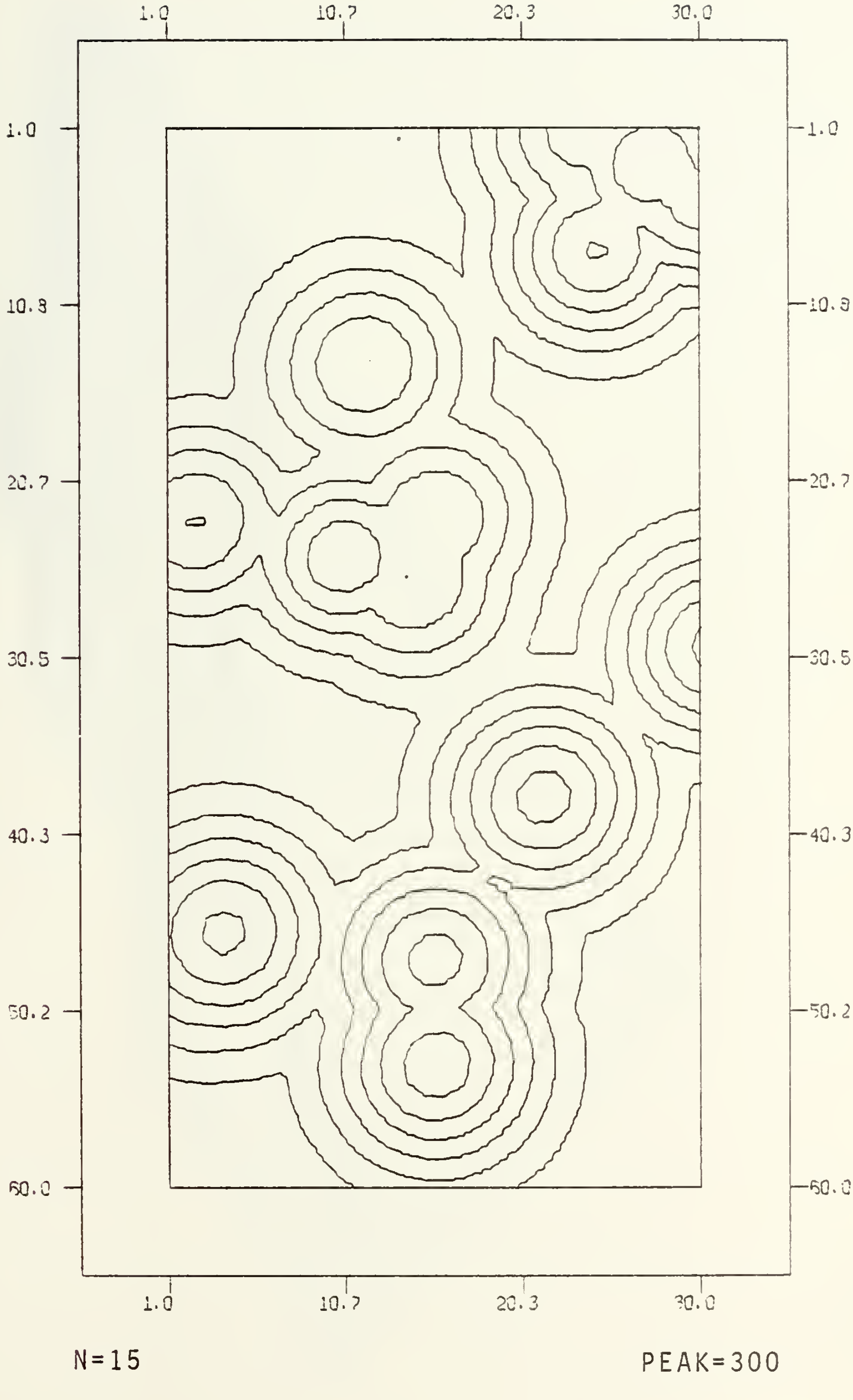

1.0
10.7
20.3
30.0
10.8
20.7
30.5
40.3
50.2
60.0
N=5
PEAK=300

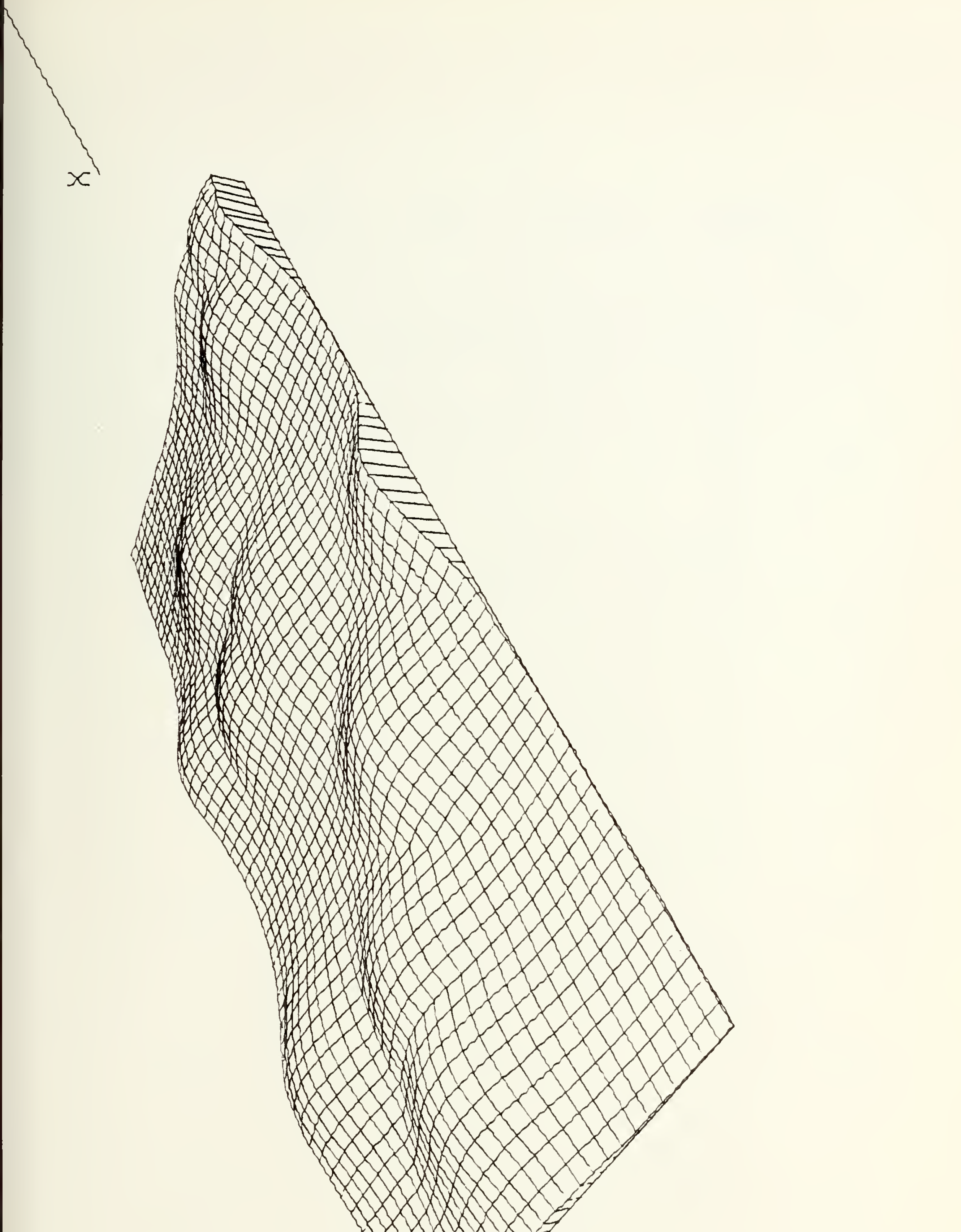

X
TERRAIN
NEEDELS

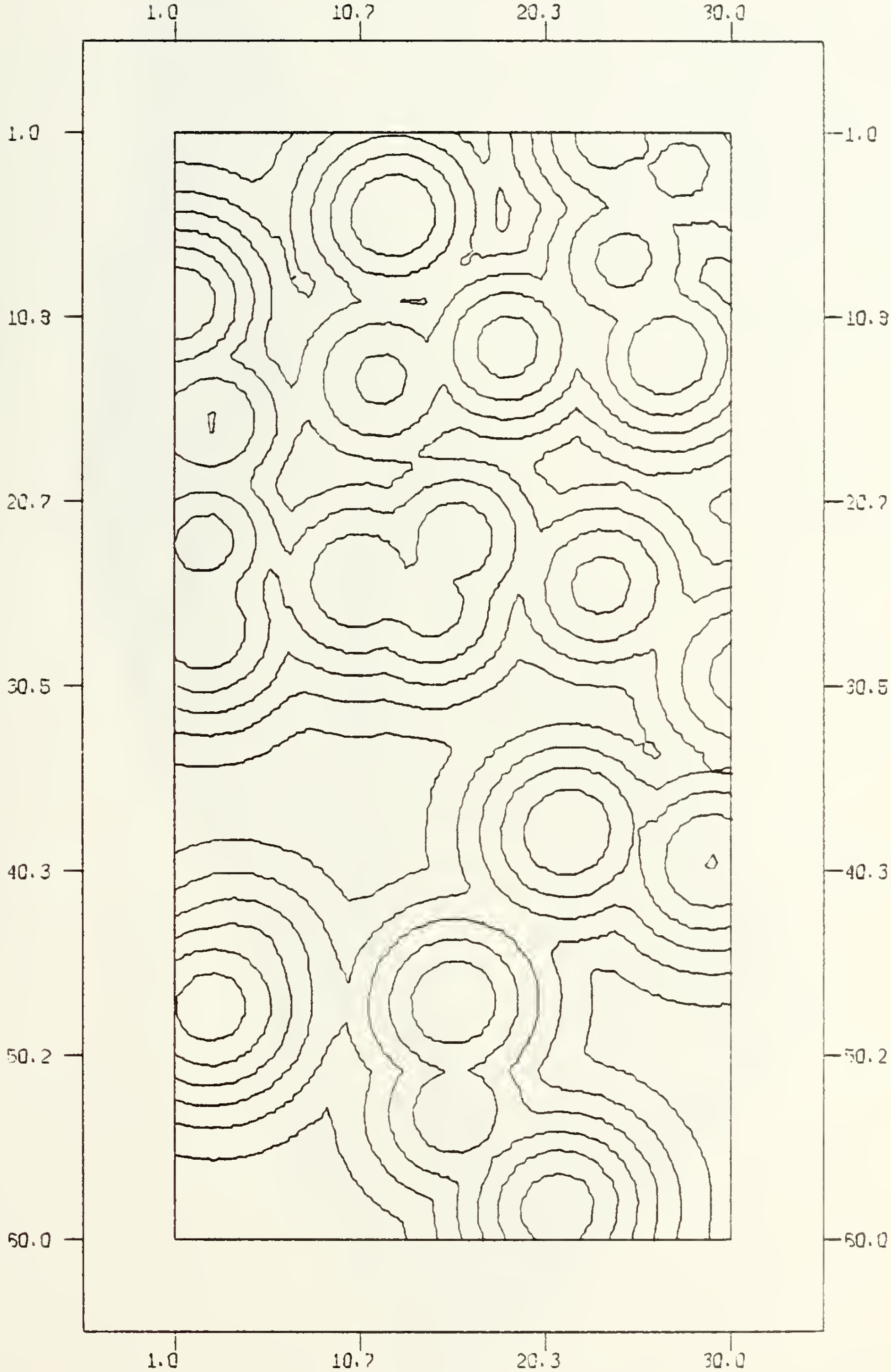


N=15

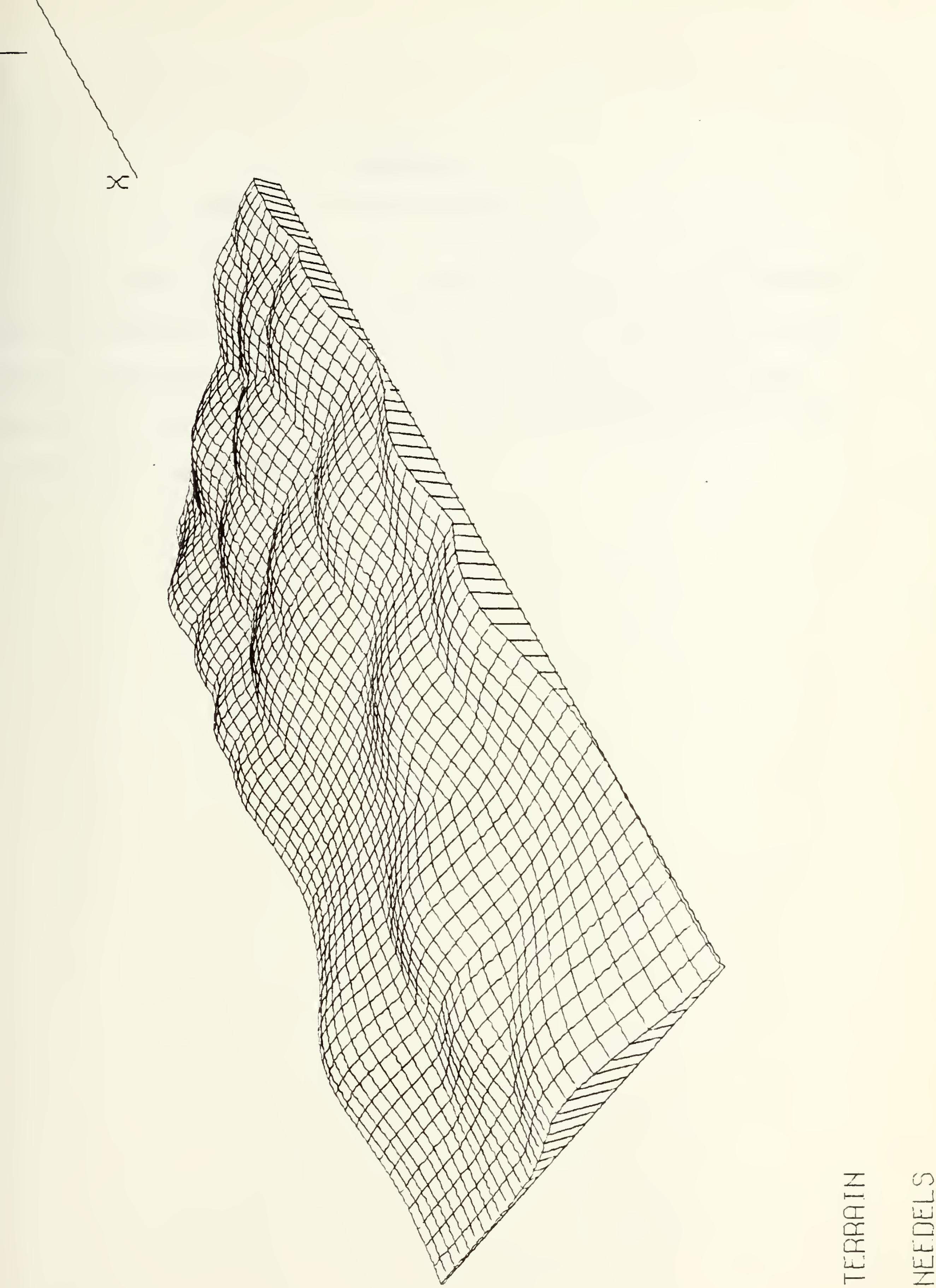
PEAK=300


1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0
N=15
PEAK=300


X
TERRAIN
NEEDELS



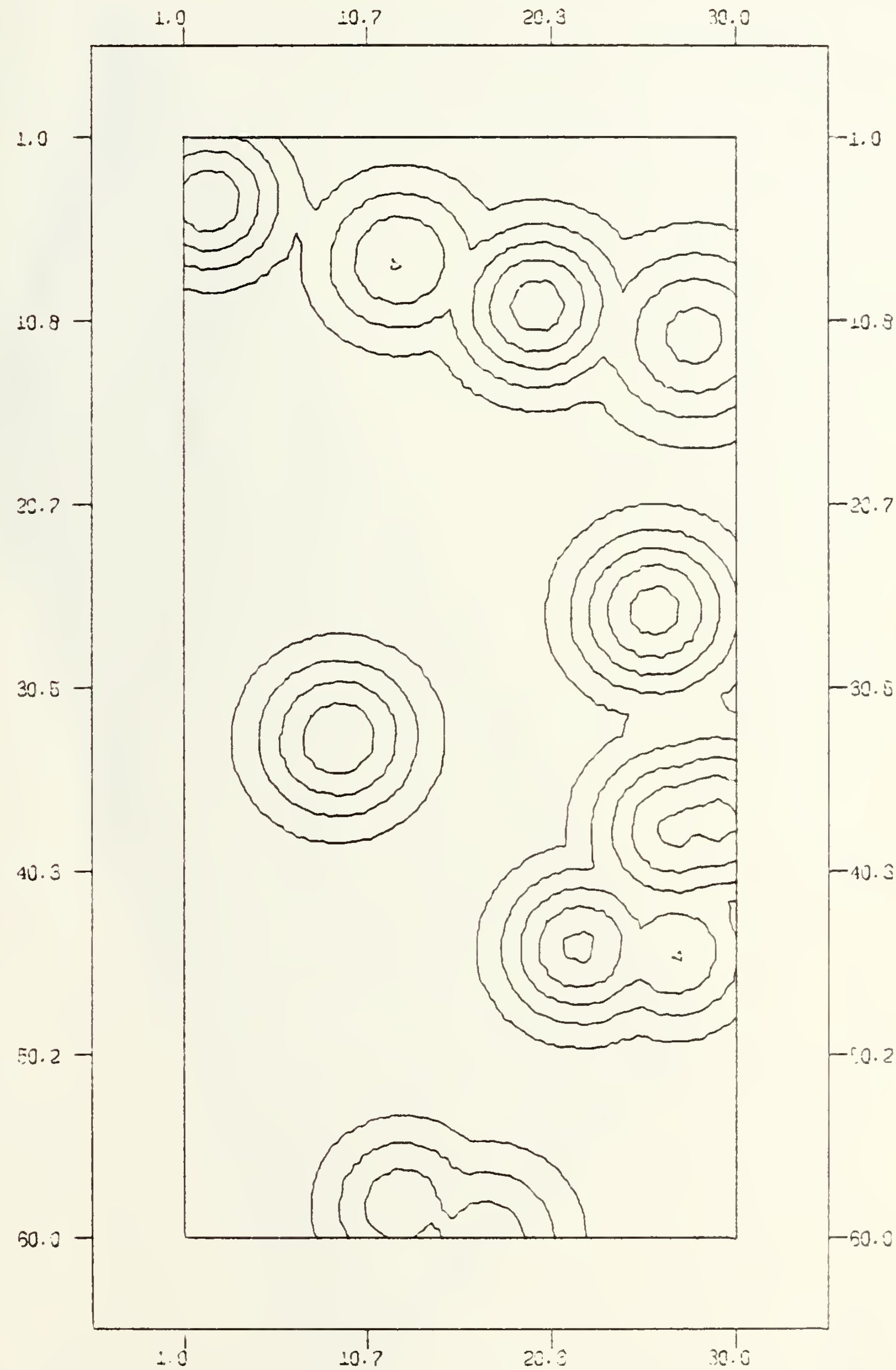
N=25 PEAK=300

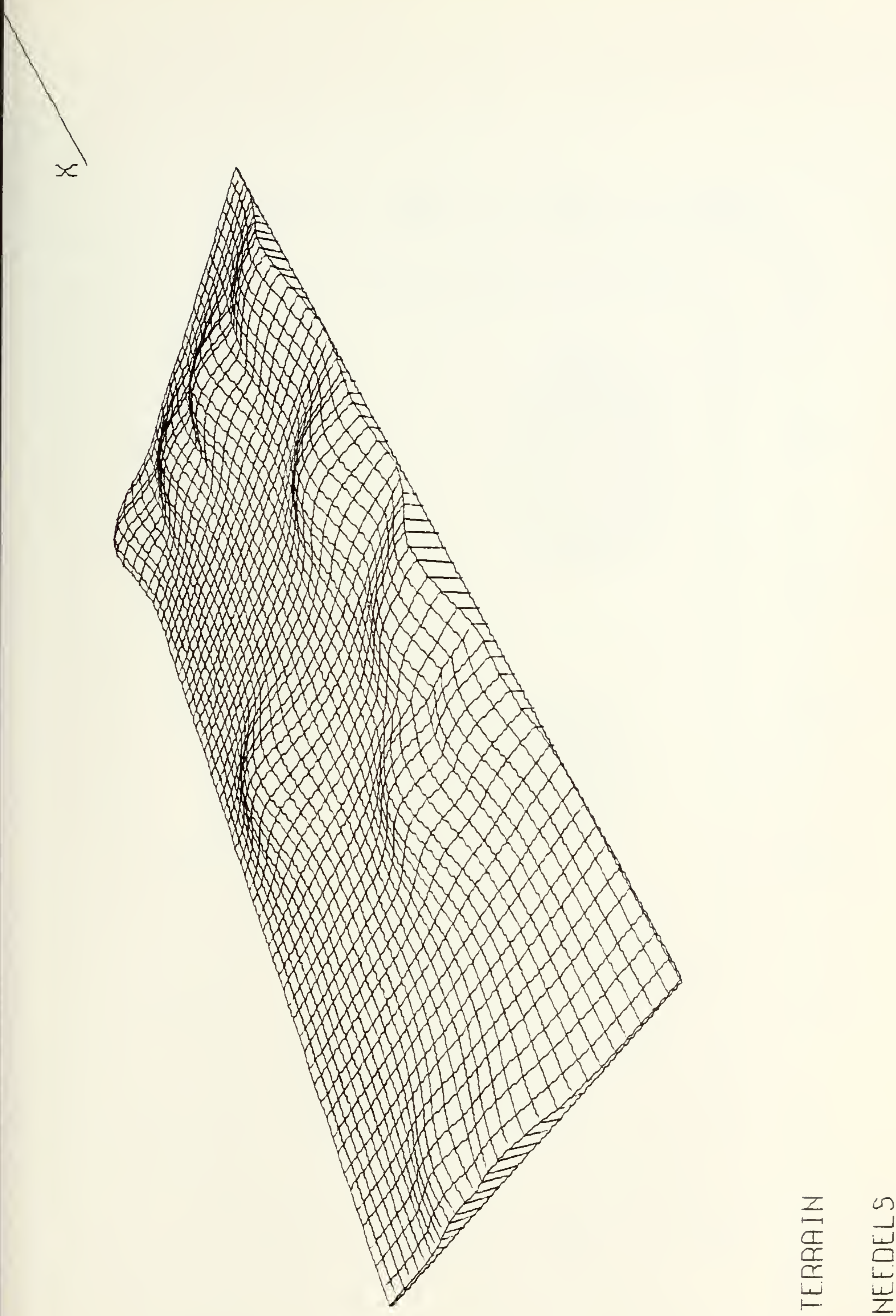

x
TERRAIN
NEEDELS

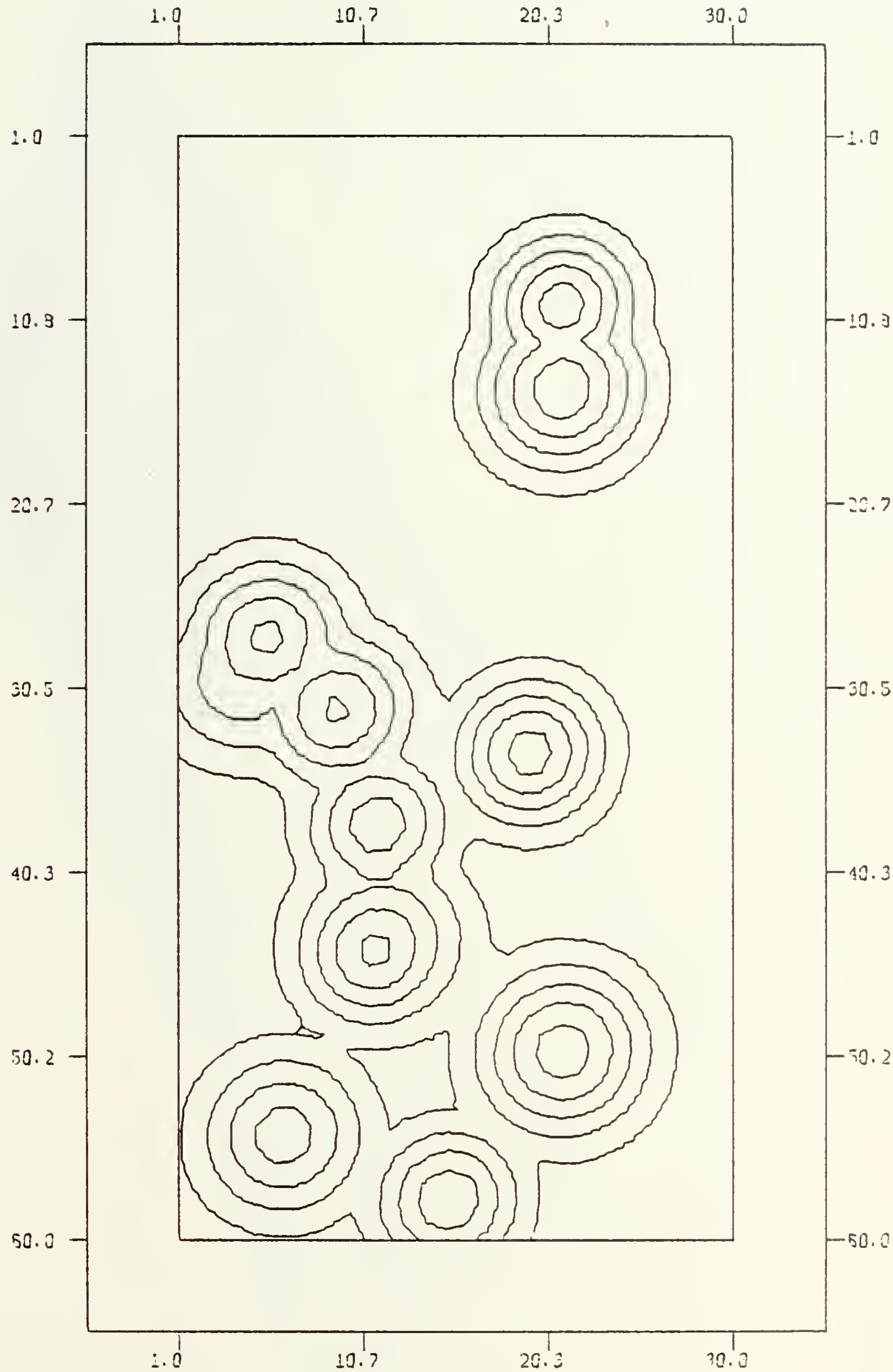
APPENDIX B

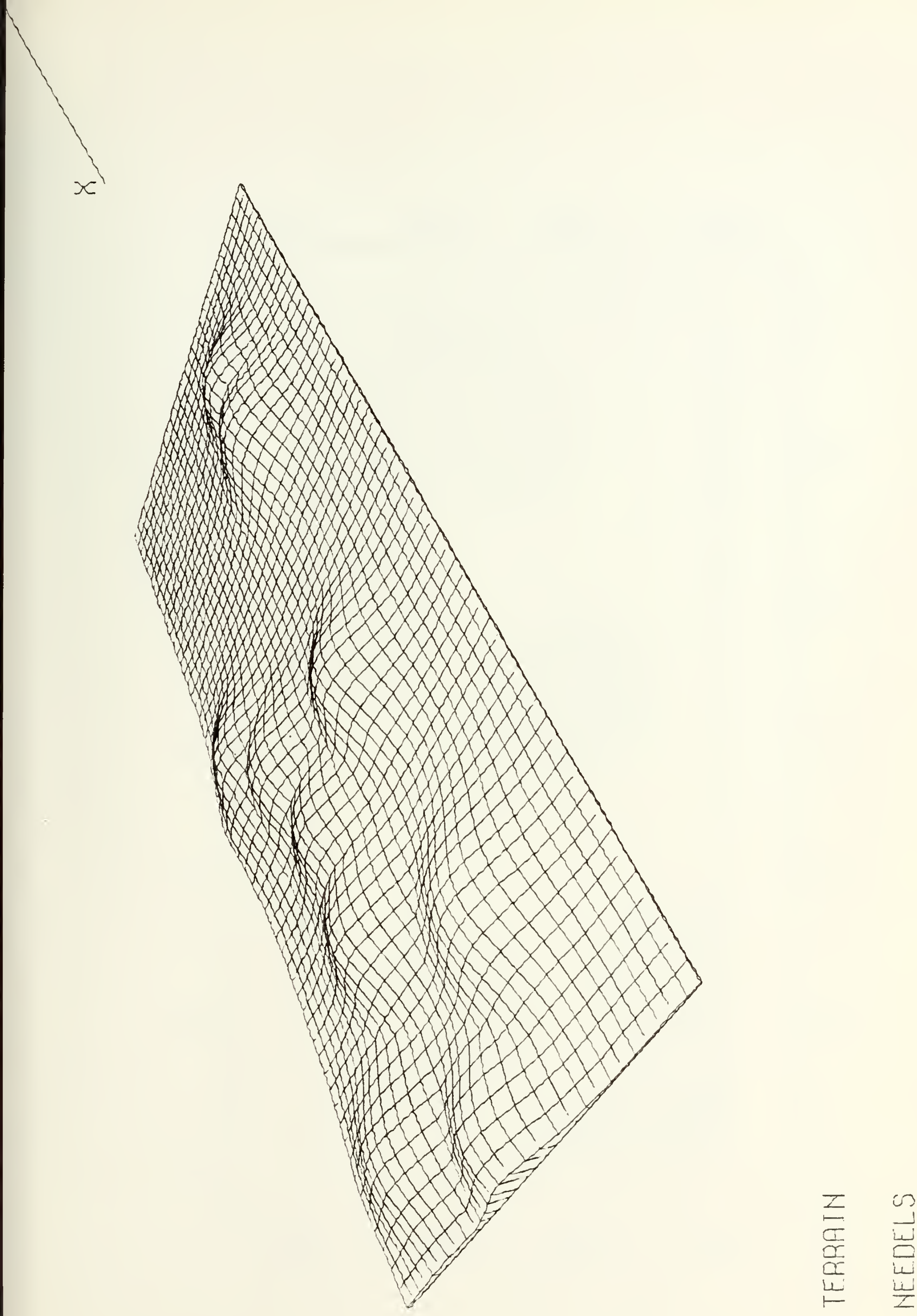
VARIABLE RANDOM NUMBER SEEDS

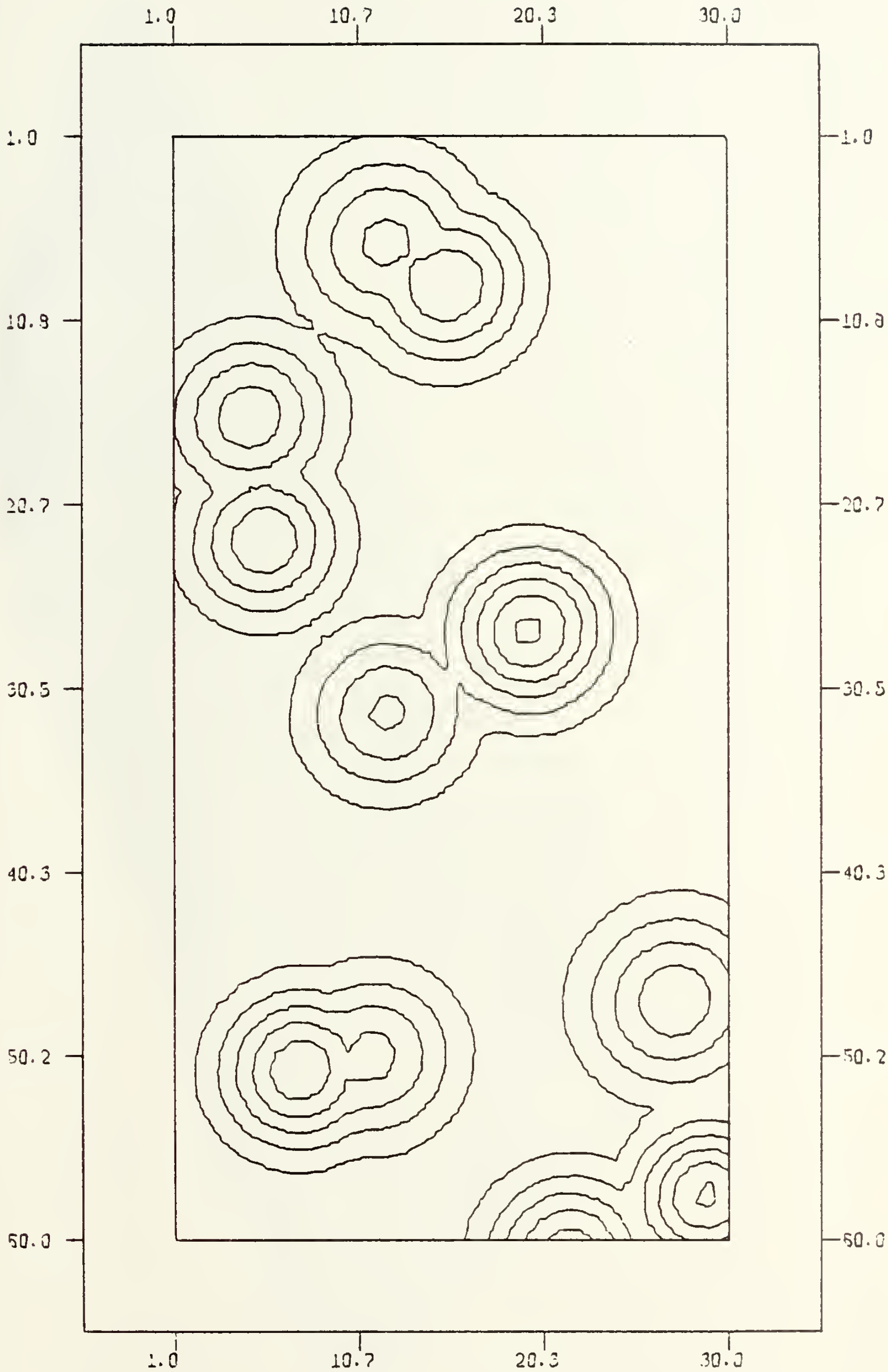
The graphs included in this appendix have the same constant input parameters. The only variable is the seed for the random number generator. As with the previous appendix, the graphs appear in pairs; the contour map first, followed by its three dimensional representation.

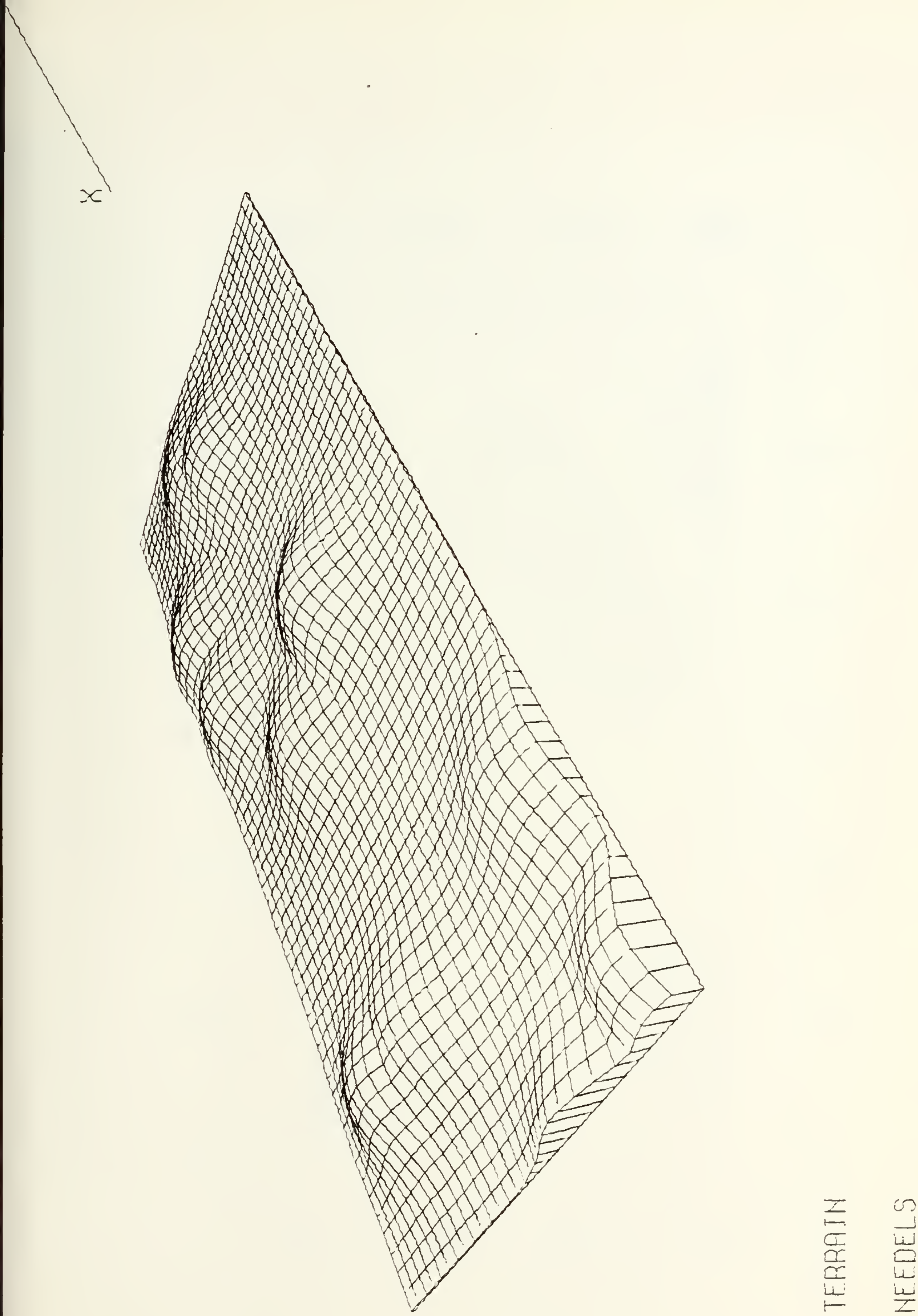

1.0
10.7
20.3
30.0
10.8
20.7
30.5
40.3
50.2
60.0

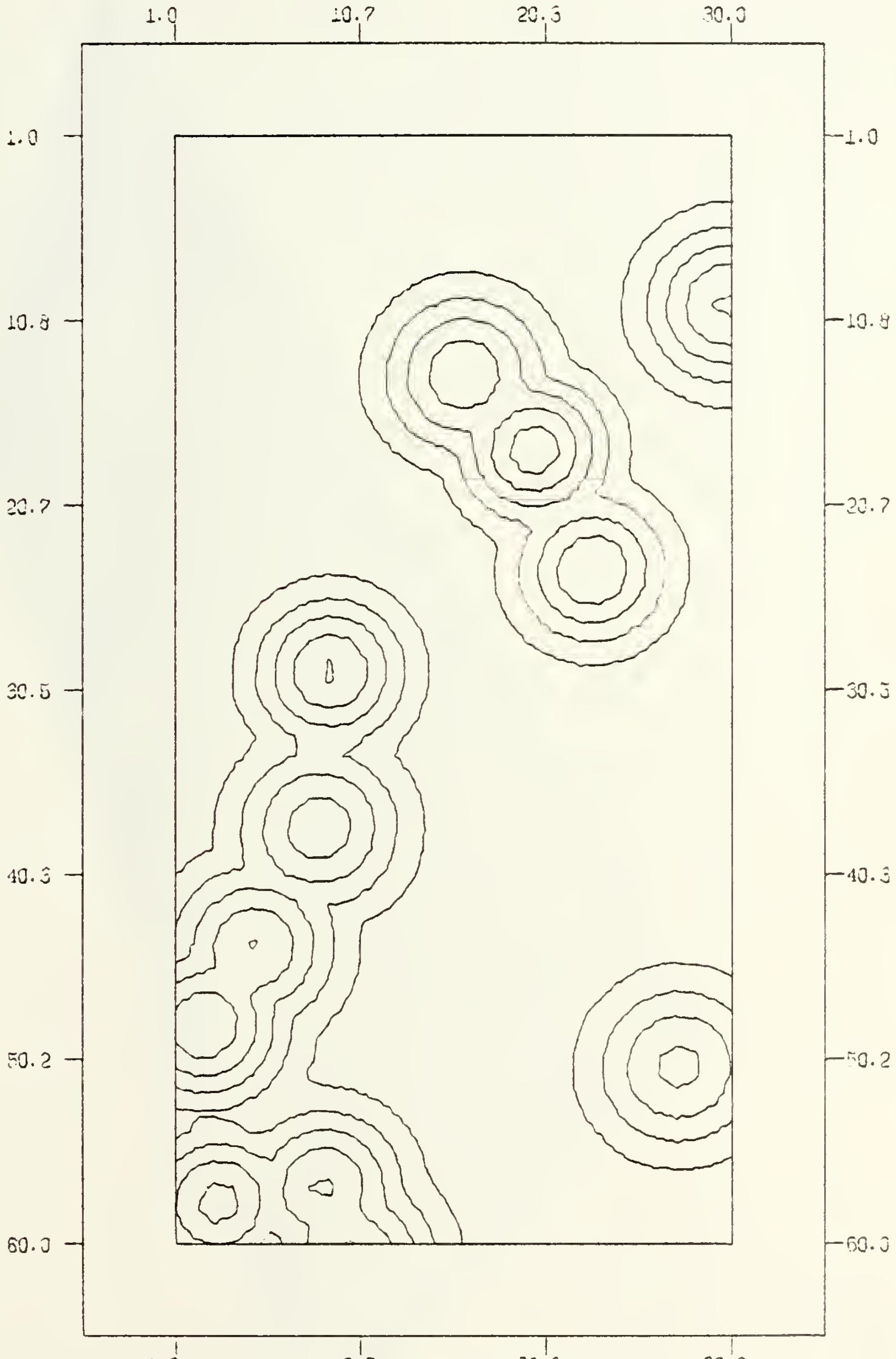

x
TERRAIN
NEEDELS

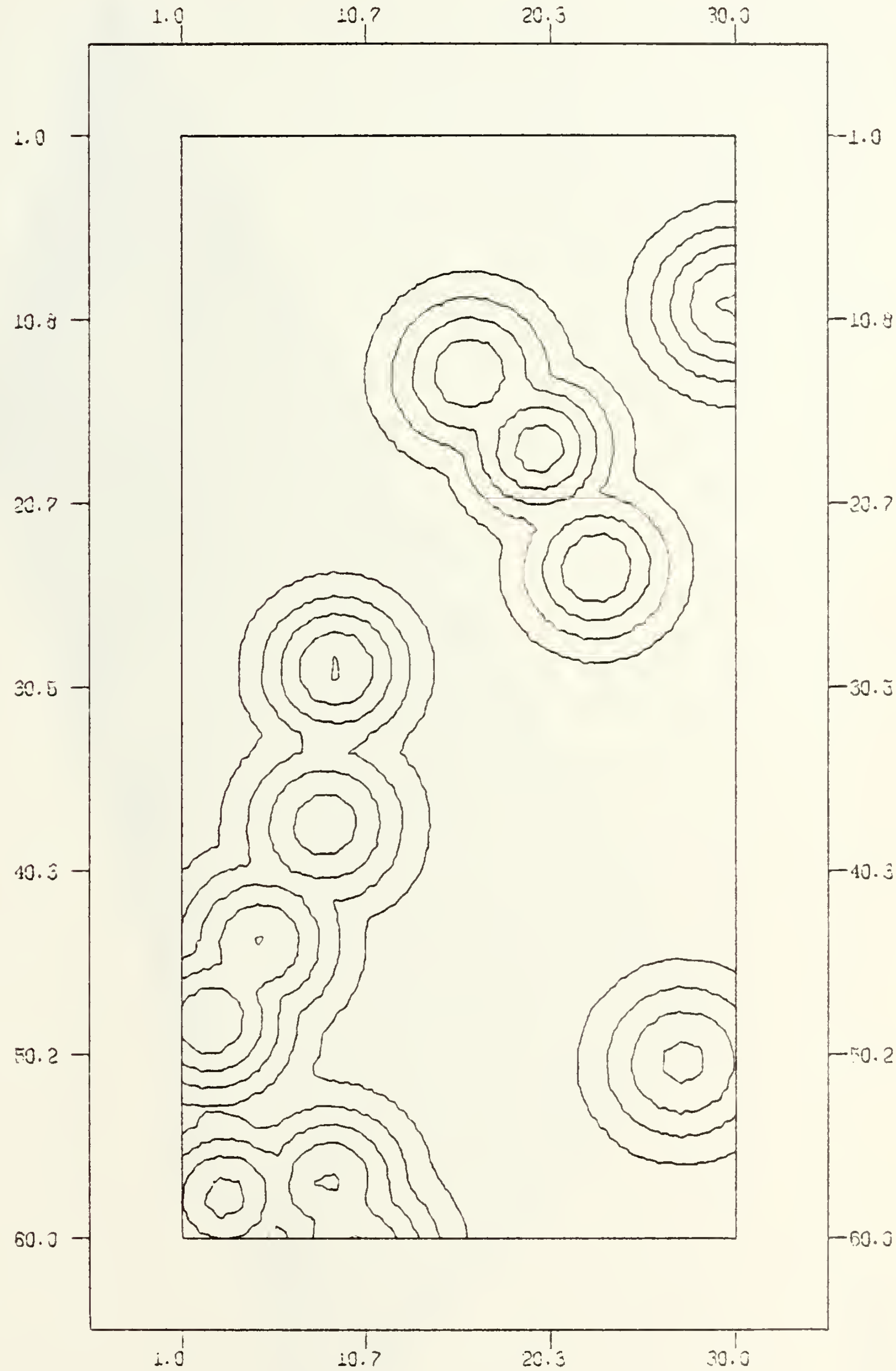

1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0


x
TERRAIN
NEEDELS

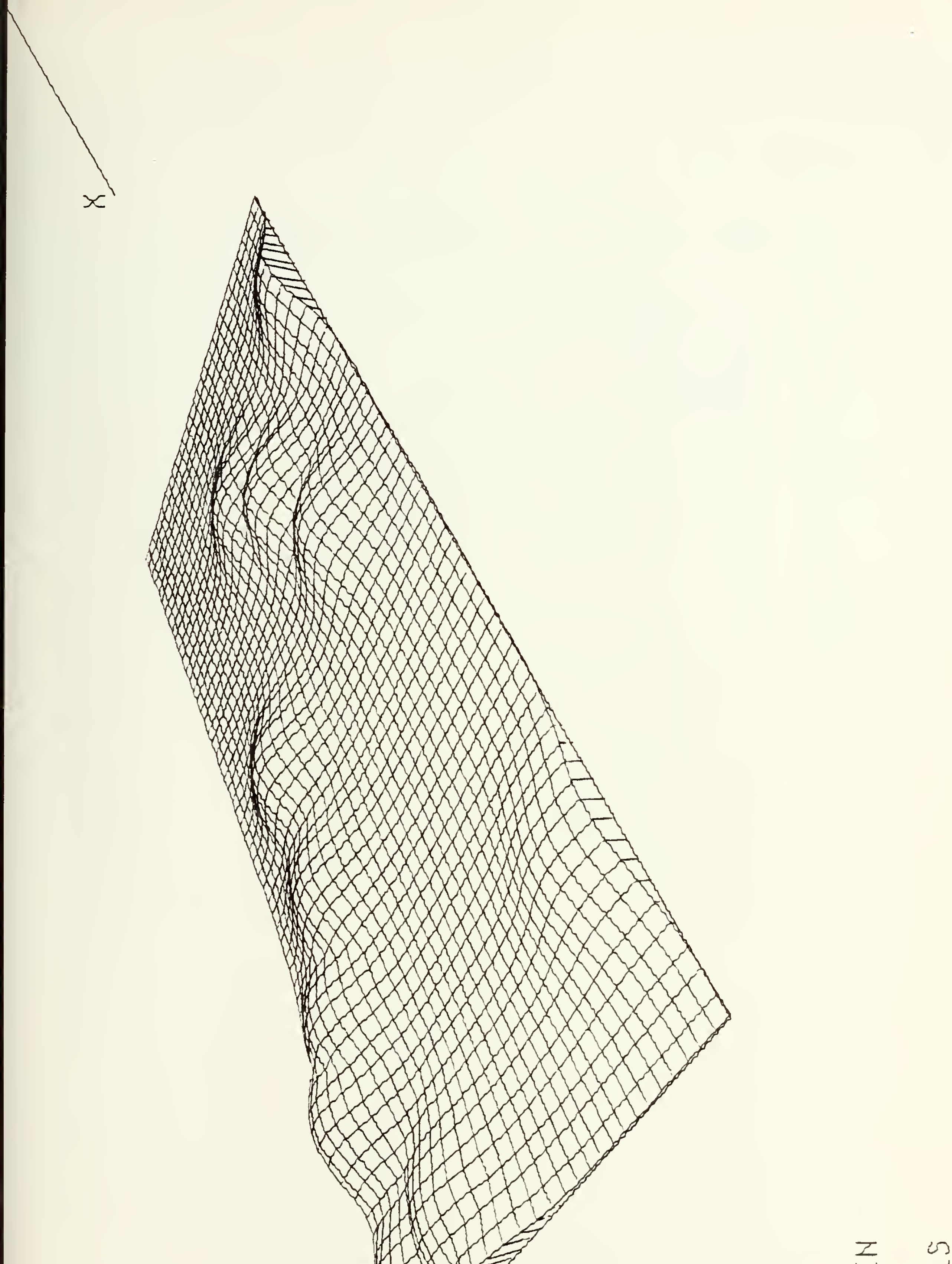

1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0

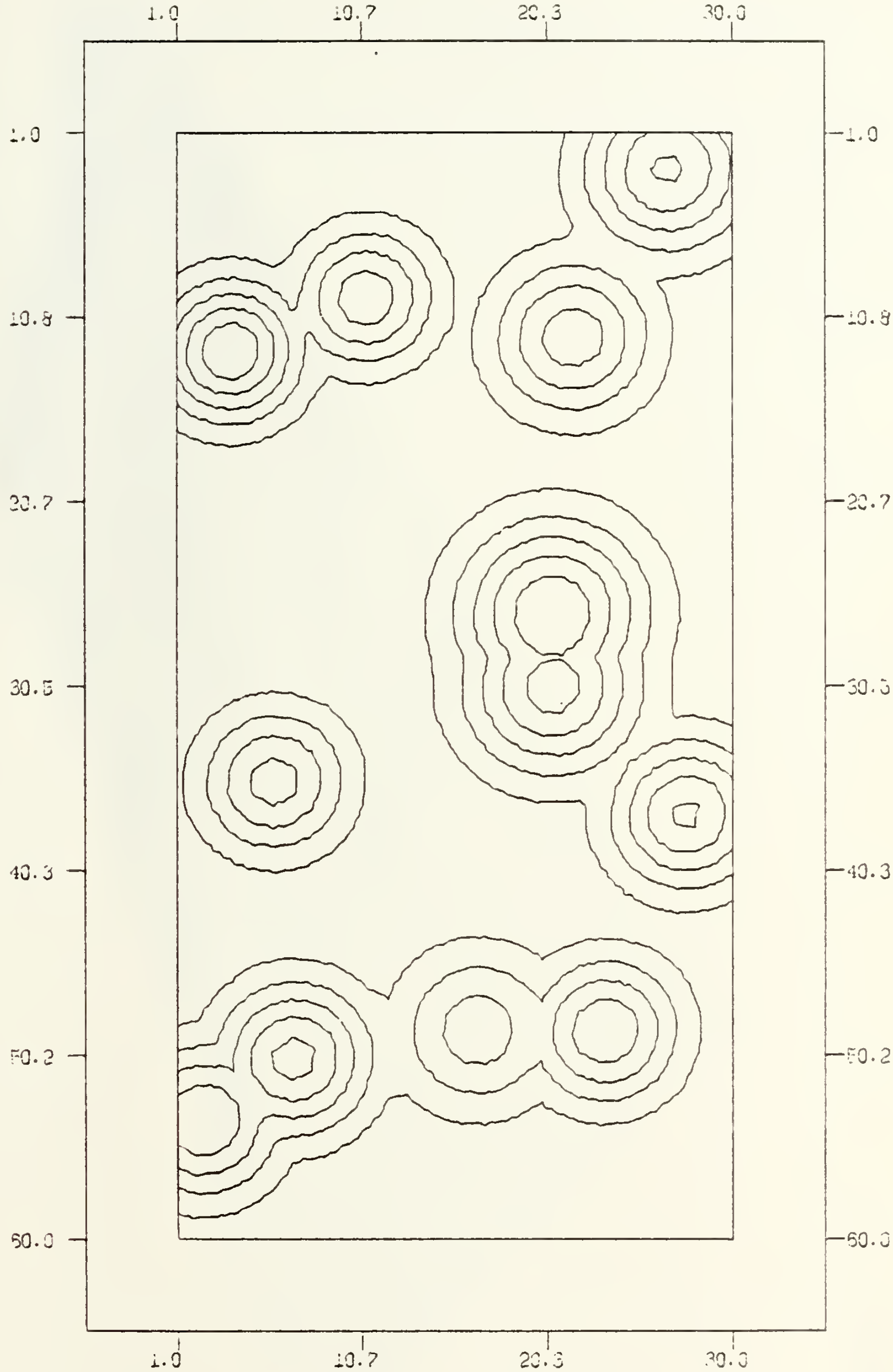

x
TERRAIN
NEEDELS

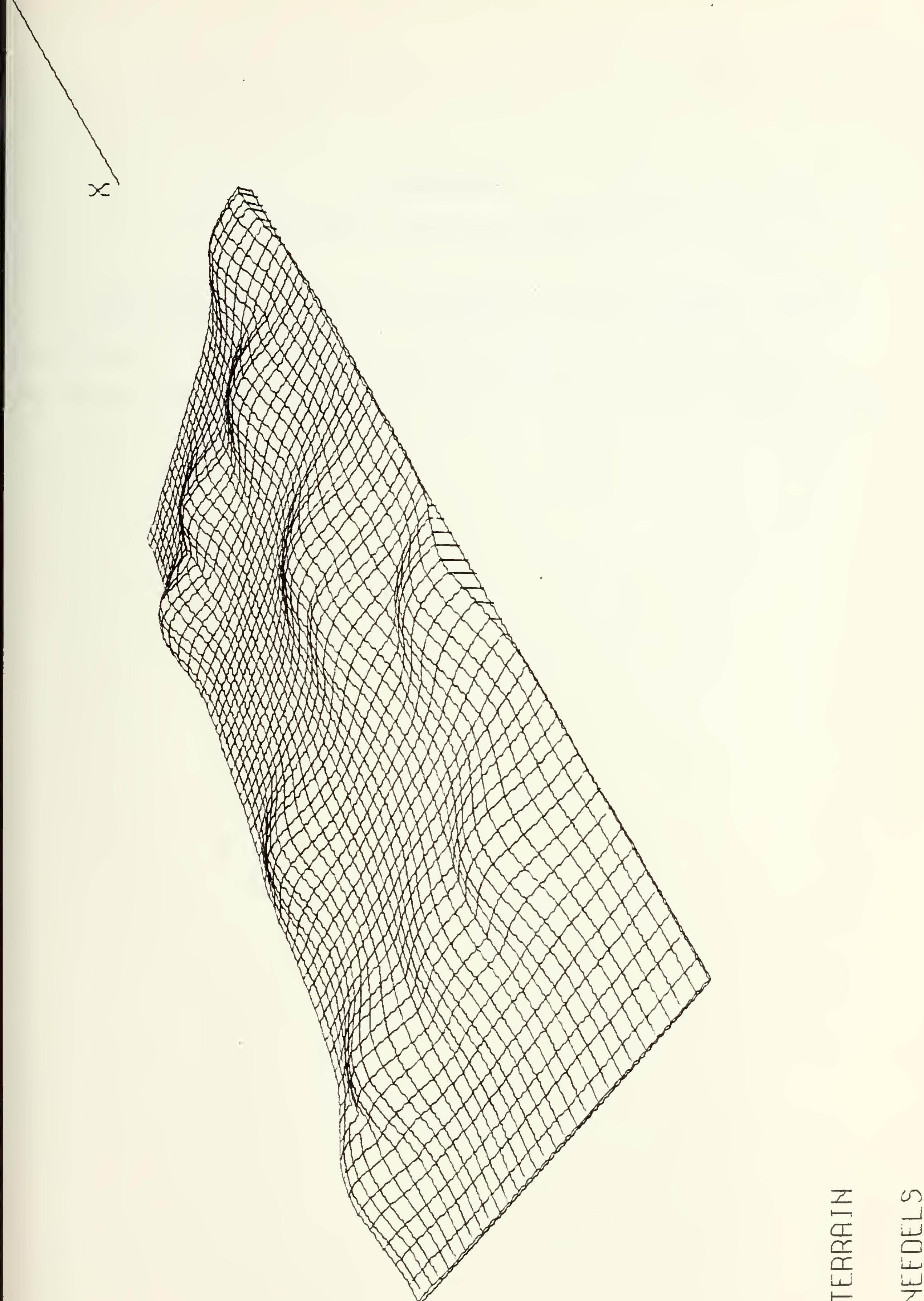

1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0



1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0


X
TERRAIN
NEEDELS

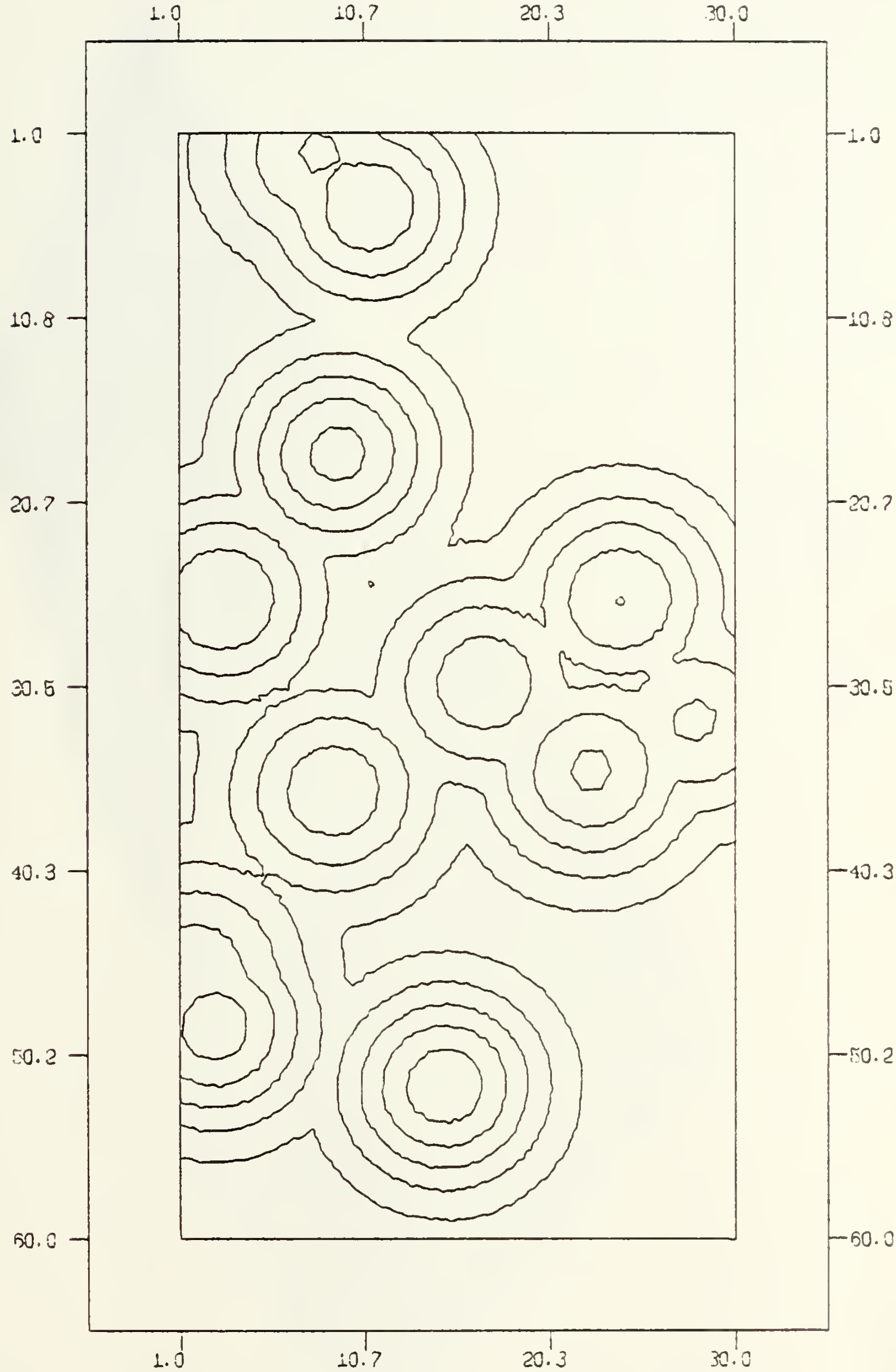

1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0

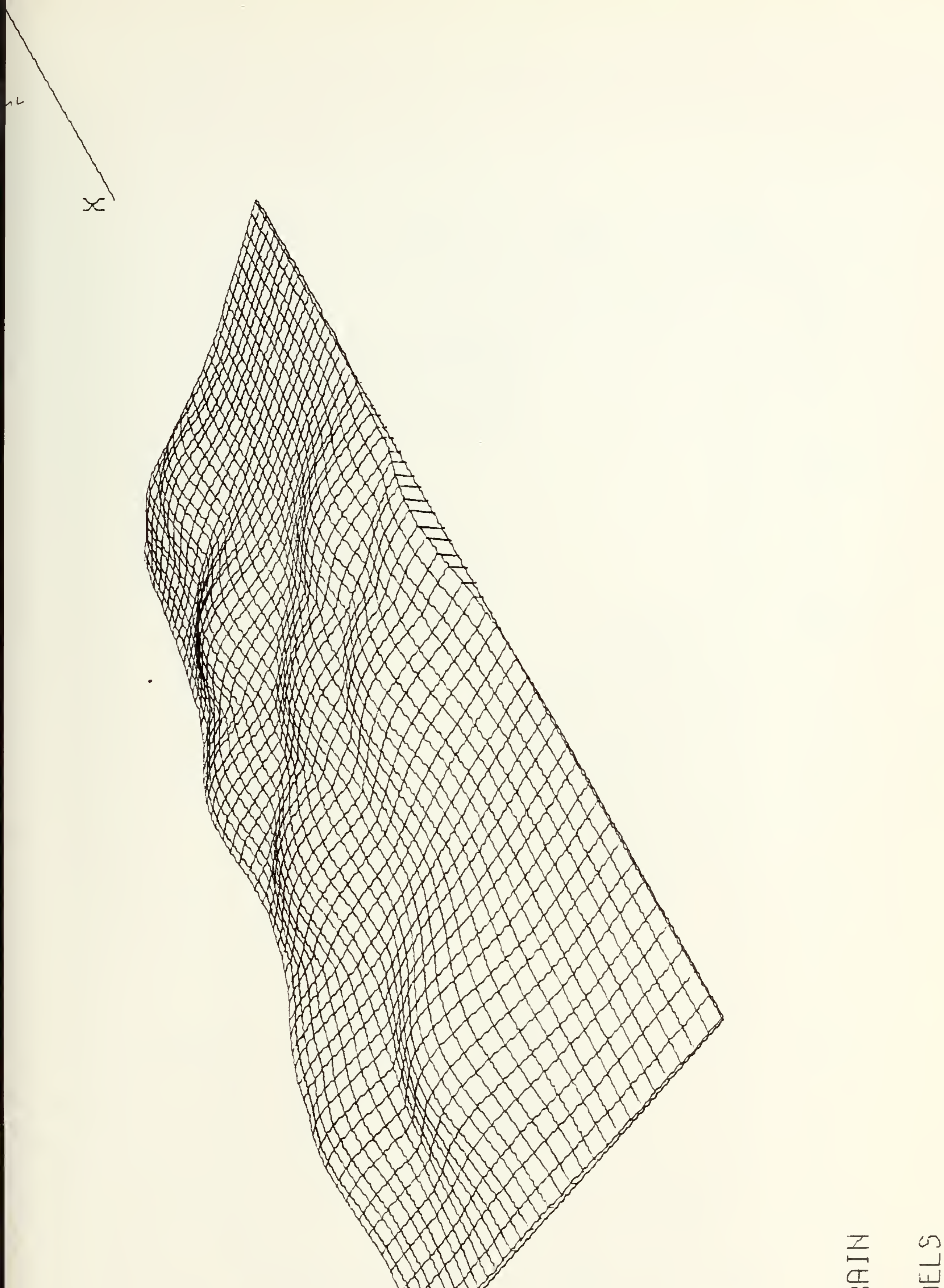

X
TERRAIN
NEEDELS

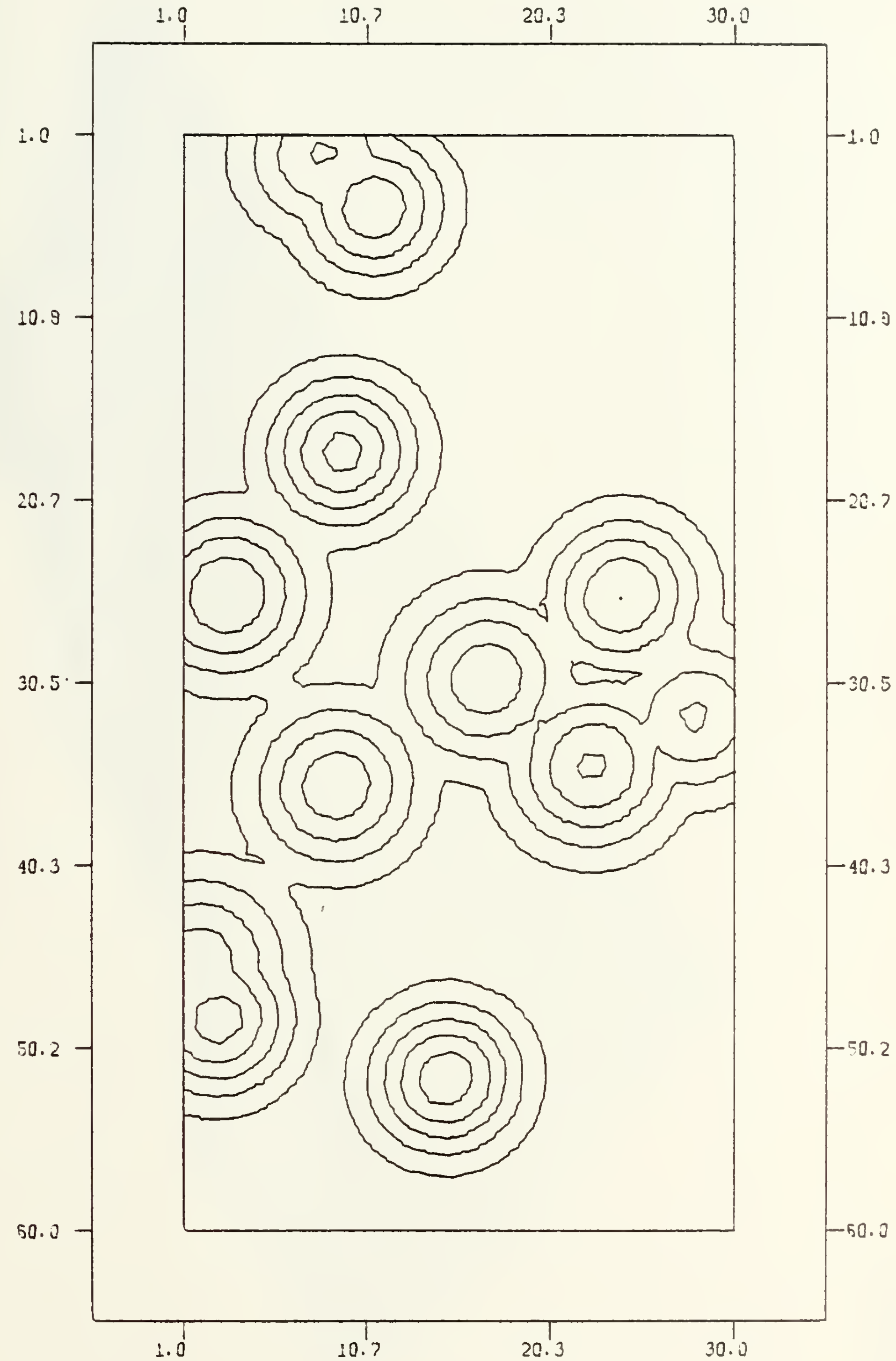
APPENDIX C

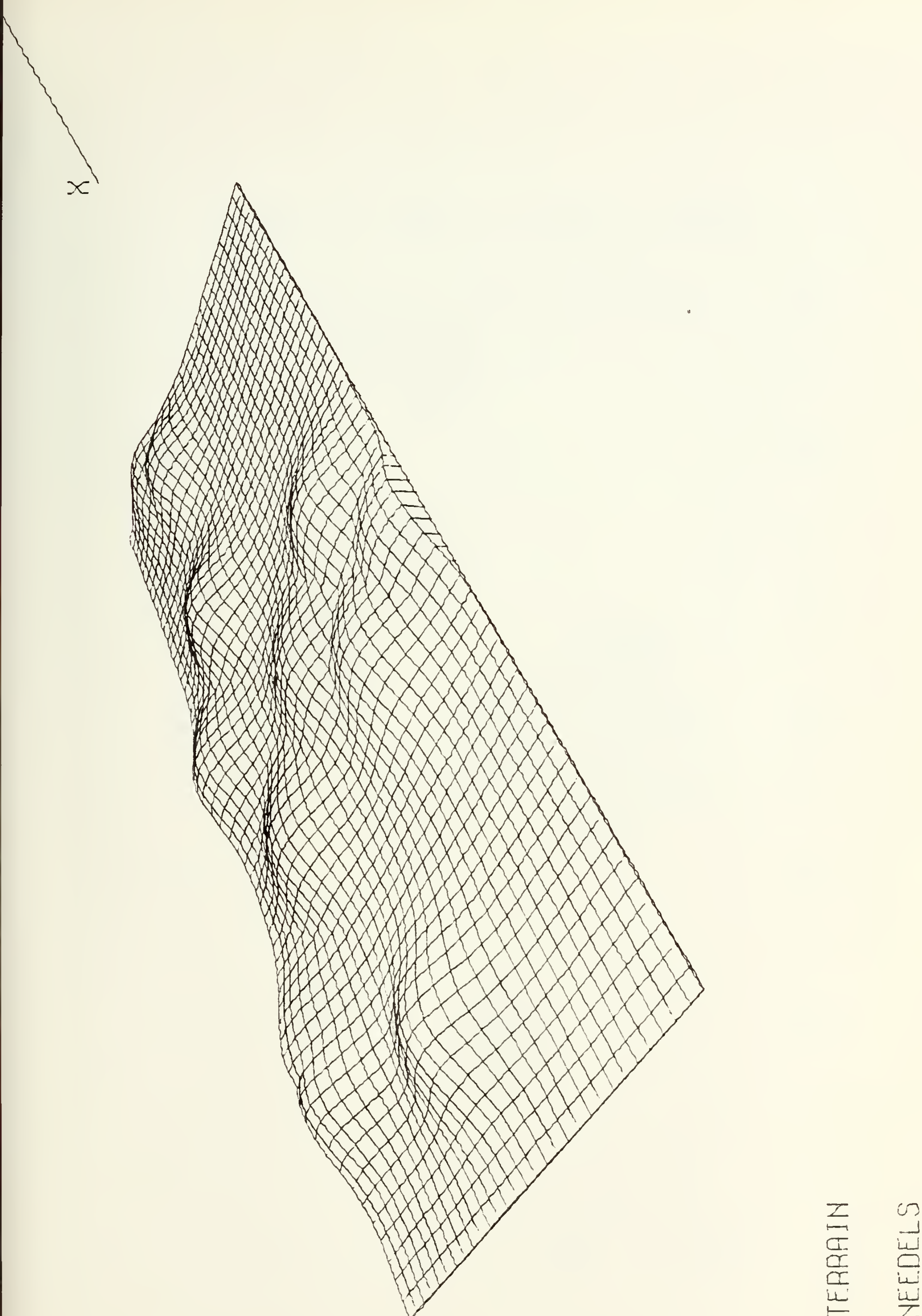
VARIABLE SPREAD OF HILLS

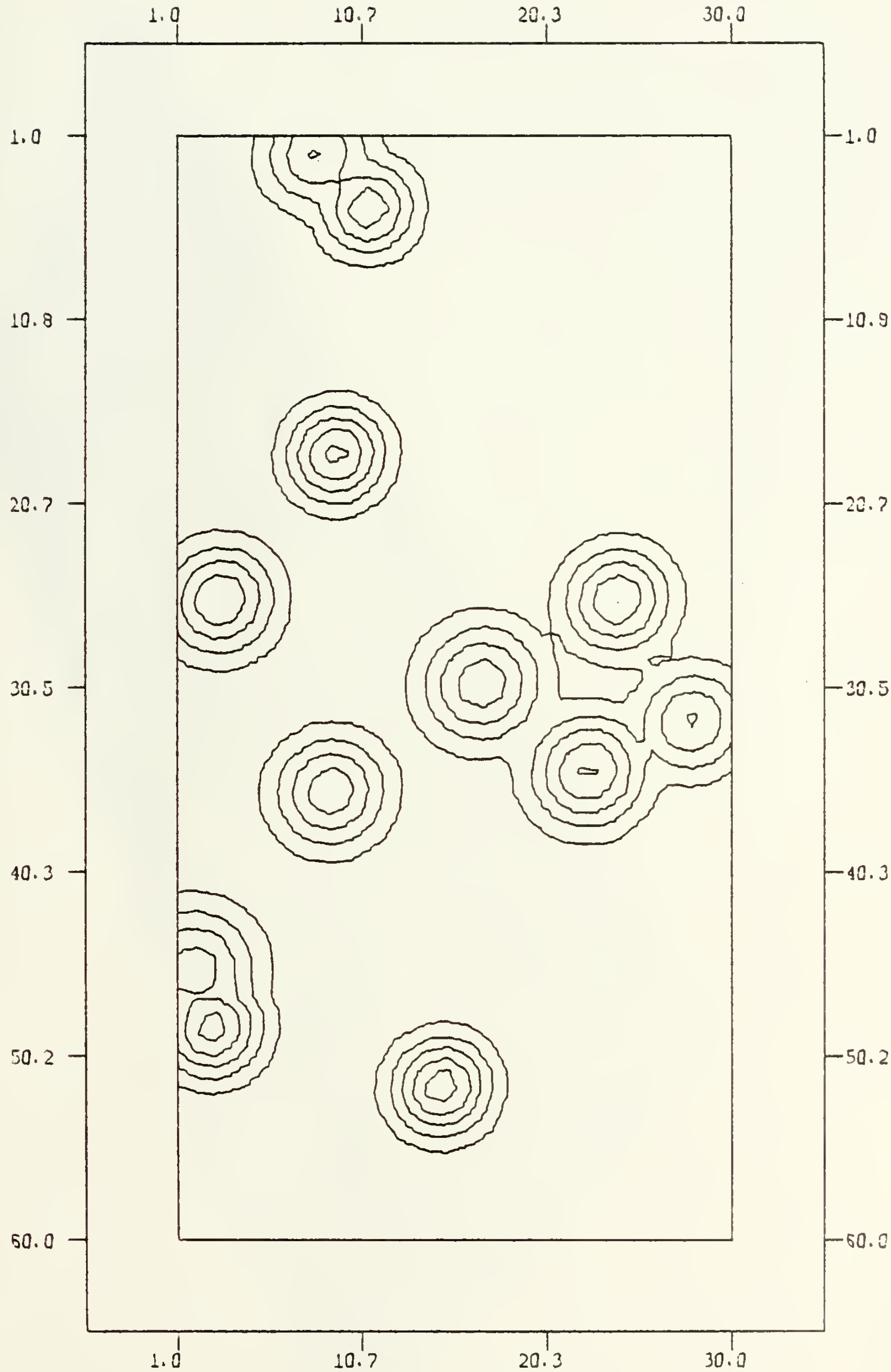
The graphs presented in Appendix C have constant input parameters except for the spread of the hills (XMEL,YMEL). The three variations are 100,200, and 300 meters, respectively.

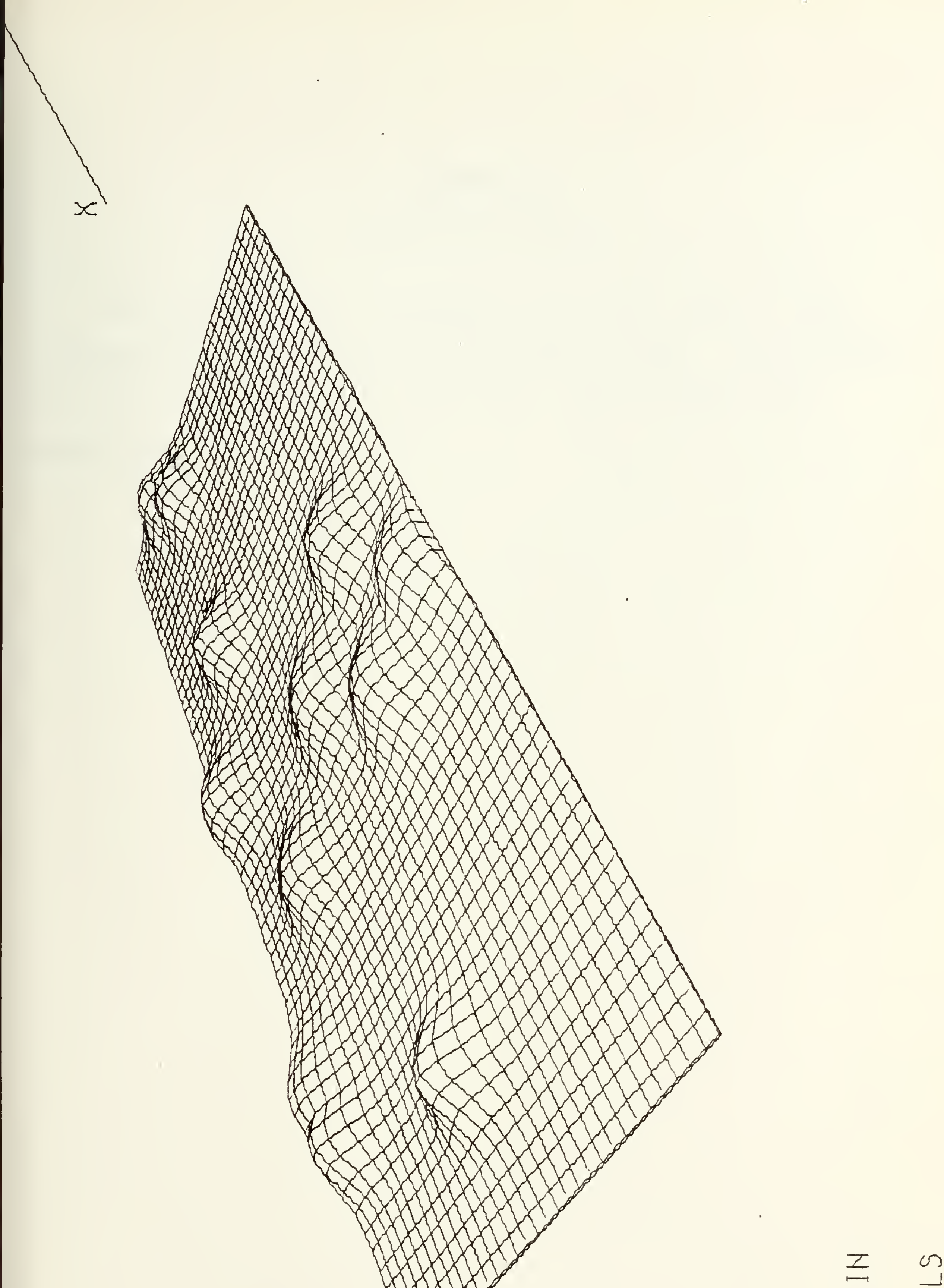

1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0


X
TERRAIN
NEEDELS


1.0
10.7
20.3
30.0
1.0
10.9
20.7
30.5
40.3
50.2
60.0


X
TERRAIN
NEEDELS

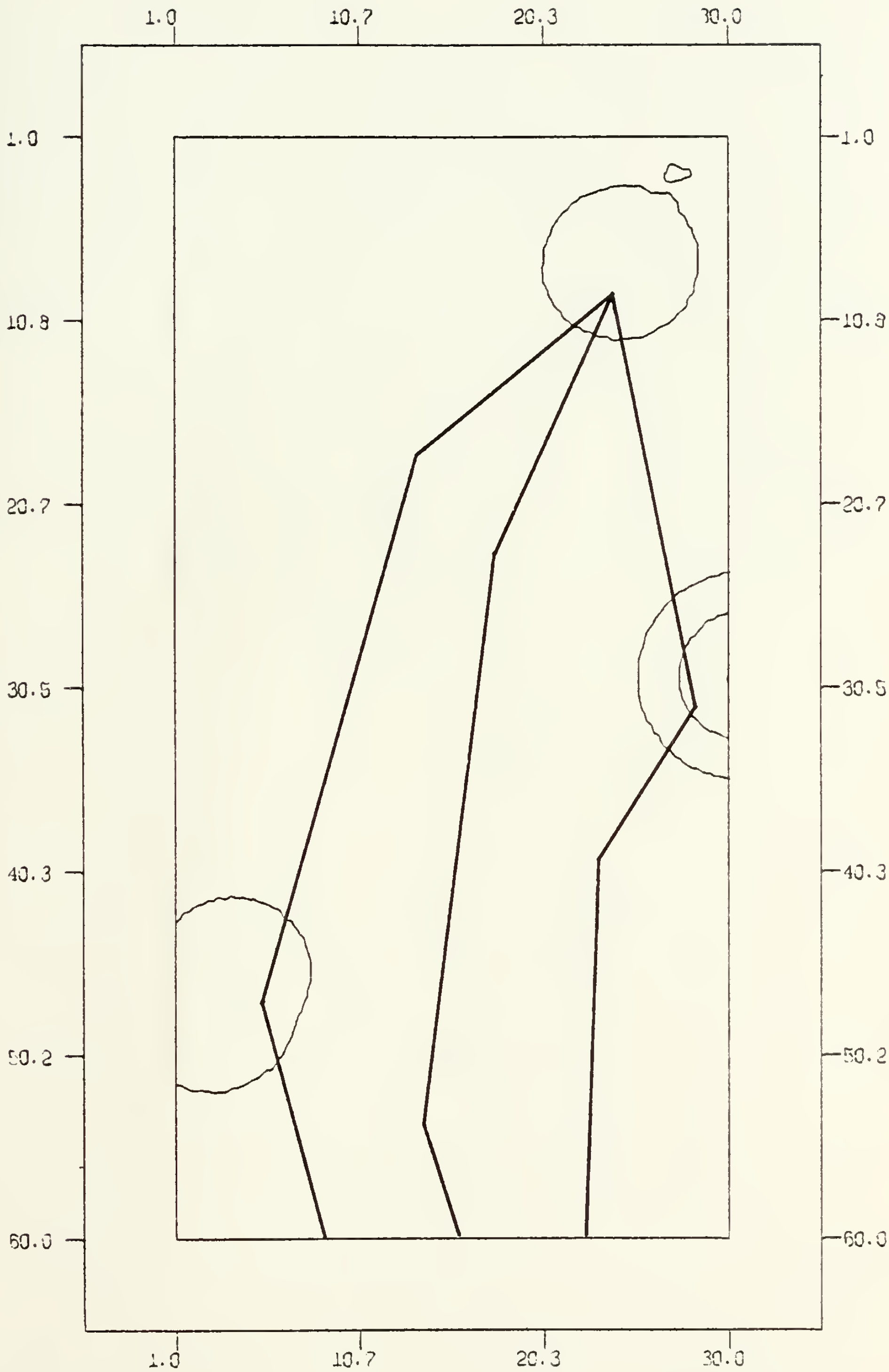

1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0
1.0
10.9
20.7
30.5
40.3
50.2
60.0
1.0
10.7
20.3
30.0


X
TERRAIN
NEEFDELS

APPENDIX D

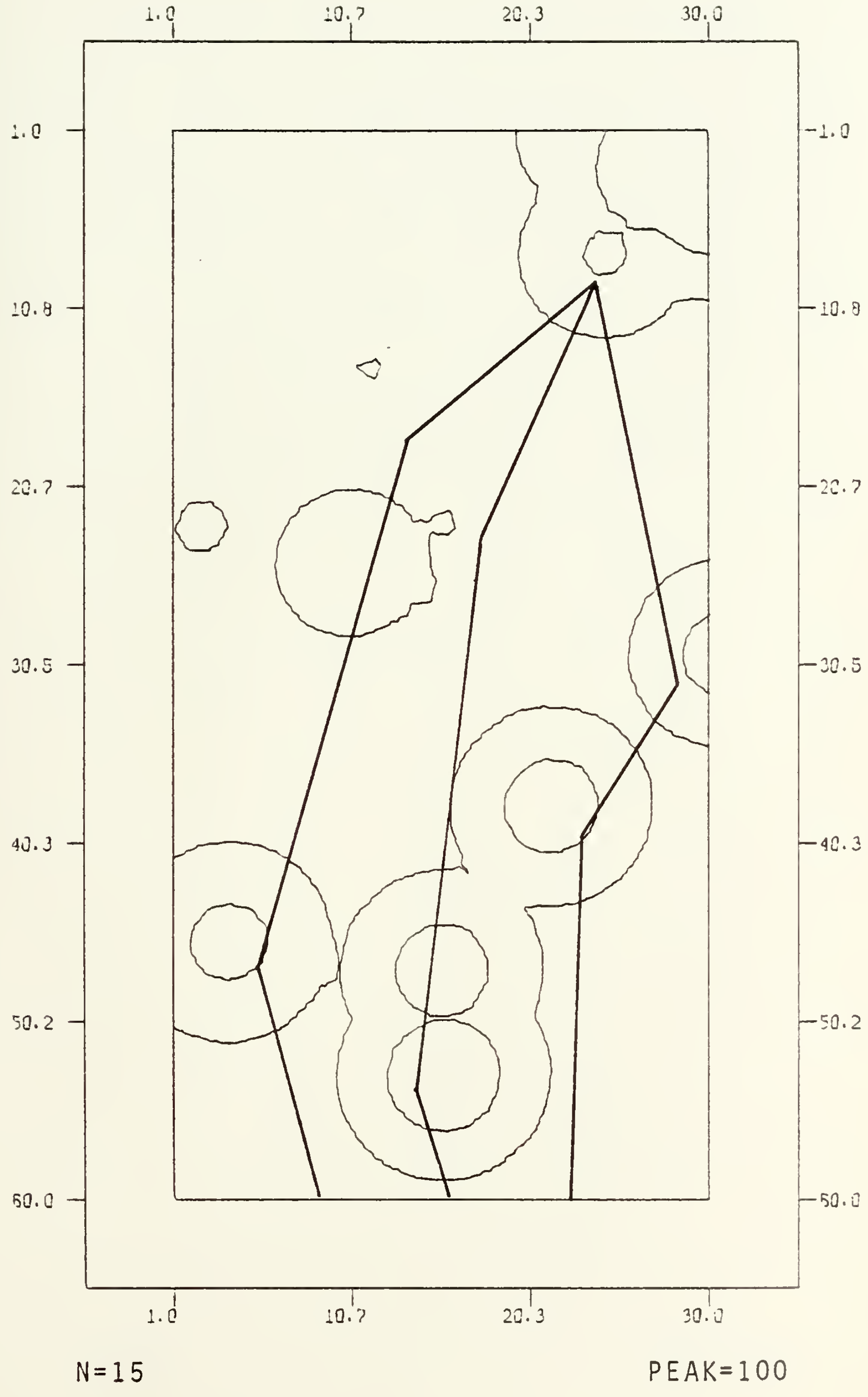
ROUTE SELECTION

The contour maps in Appendix D are the same as those in Appendix A except for the overlayed routes. These graphs were used to develop input data for the line-of-sight subroutine.

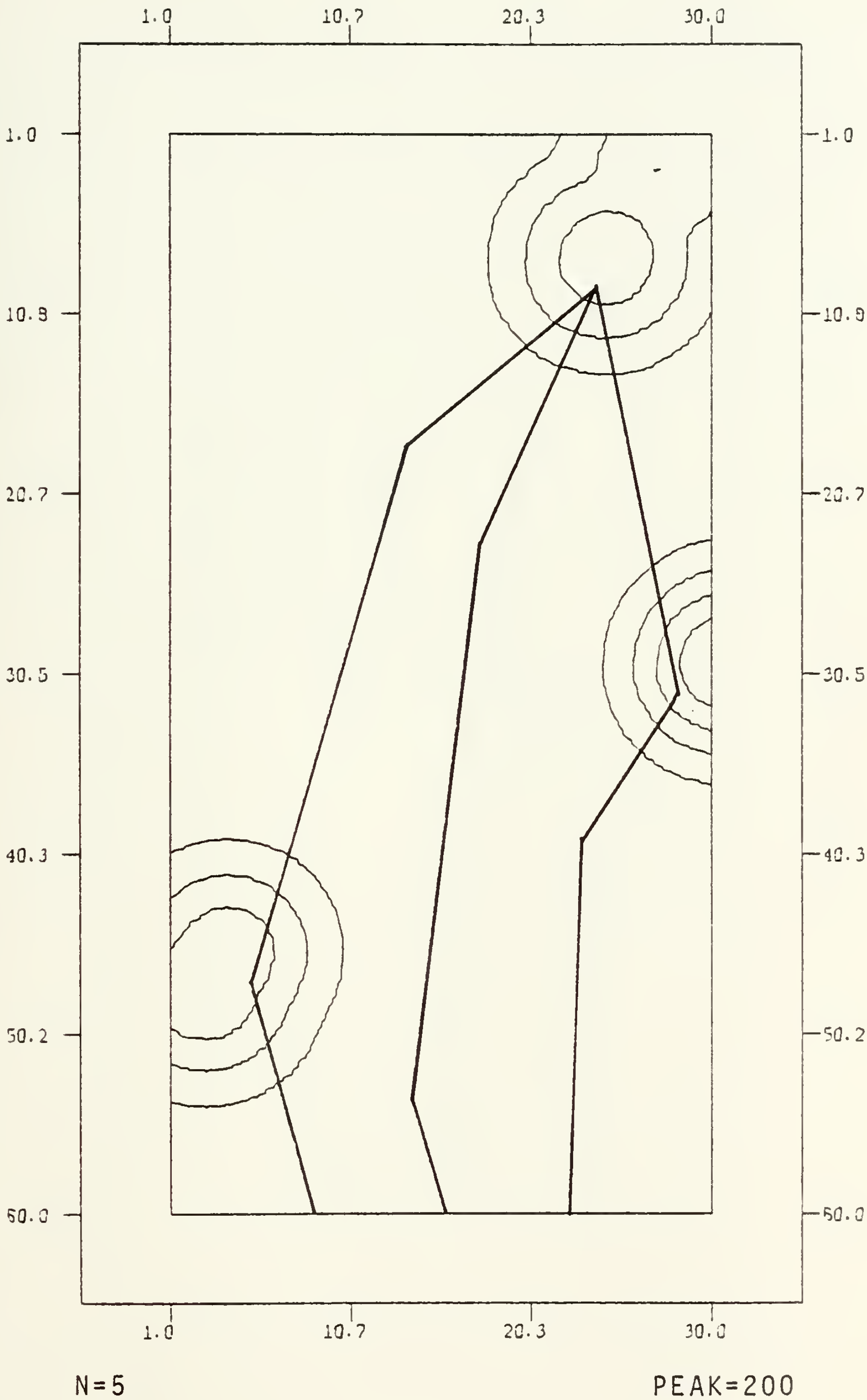


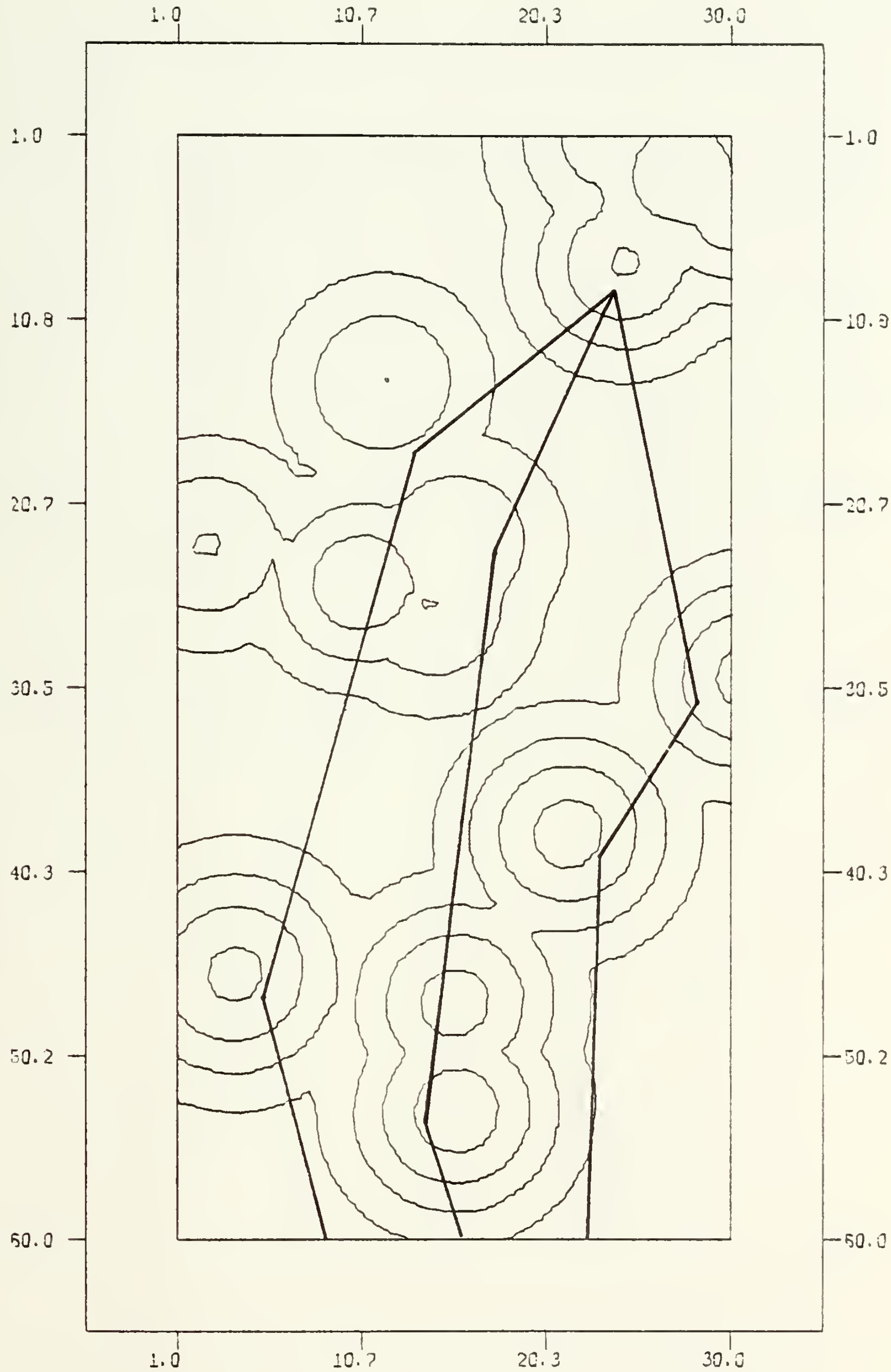
N=5

PEAK=100

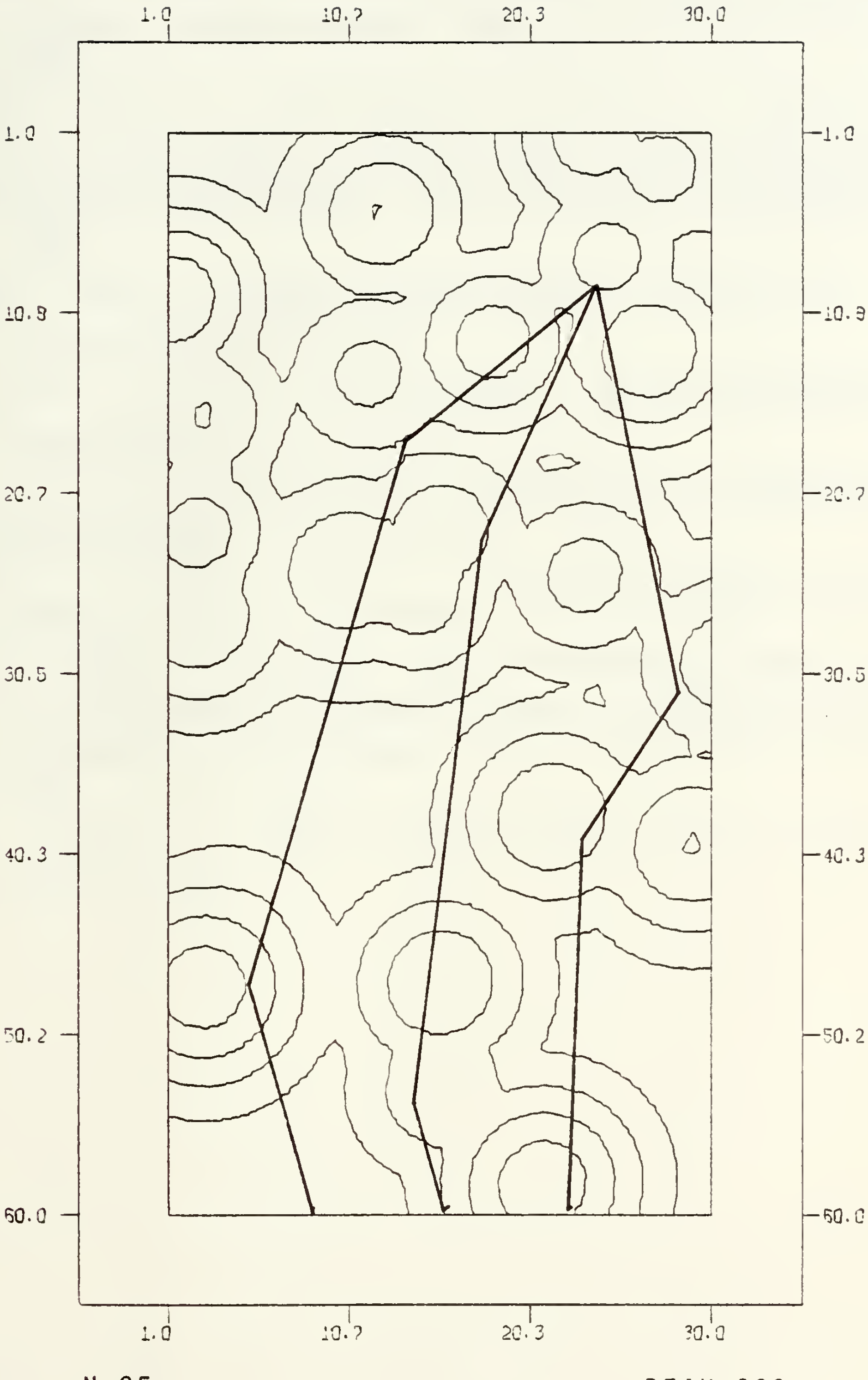

1.0
10.7
20.3
30.0
10.8
20.7
30.5
40.3
50.2
60.0
N=15
PEAK=100


1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0
N=25
PEAK=100


1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0
N=5
PEAK=200



N=15 PEAK=200


1.0
10.7
20.3
30.0
1.0
10.8
20.7
30.5
40.3
50.2
60.0
N=25
PEAK=200

BIBLIOGRAPHY

Bishop, Albert B., and Clark, Gordon M., The Tank Weapon System, Systems Research Group, Ohio State University, 1969.

Farrell, Robert L., and Freedman, Richard J., Investigation of the Variation of Combat Model Predictions with Terrain Line F Sight, Vector Research, Incorporated, 1975.

Graves, James W., Tank Exchange Model, Insitute for Defense Analyses, 1973.

Luenberger, David G., Introduction to Linear and Nonlinear Programming, Addison-Wesley, 1973.

Meyer, Paul L., Introductory Probability and Statistical Applications, Addison-Wesley, 1972.

Murphy, N. R., and Alvium, R. B., AMC-74 Vehicle Dynamics Module, Waterways Experiment Station, 1976.

Raney, Sharon D., PLT3D1: Three Dimensional Isometric or Perspective Off-line Plotting Sub-program with Hidden-line Elimination, Naval Postgraduate School, 1974.

Weir, Maurice D., Topics in Calculus Two, Naval Postgraduate School, 1973.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California	2
3.	Department Chairman, Code 55 Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	1
4.	Professor Samuel H. Parry Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	1
5.	MAJ Christopher J. Needels 76-77 CGSO Regular Course Fort Leavenworth, Kansas 66027	1
6.	US Army Military Personnel Center Attn: DAPC-OPD-PD-CS 200 Stovall Street Alexandria, Virginia 22332	1
7.	Vector Research, Incorporated Attn: Mr. Robert Farrell P. O. Box 1506 Ann Arbor, Michigan 48106	1
8.	US Army Armor and Engineer Board Attn: ATZK-AE-TA Fort Knox, Kentucky 40121	1
9.	Commander, US Army Training and Doctrine Command ATCD-AO Attn: Mr. James Smith Fort Monroe, Virginia 23651	1
10.	Director, TRADOC Systems Analysis Activity ATAA-tem Attn: Mr. Carrol Denny and Mr. Al Burnham White Sands Missle Range New Mexico 88002	1

11. Commander in Chief US Pacific Fleet 1
Attn: CDR Karl Eulenstein
J021 Review and Analysis Department
Box 13
FPO San Francisco, Clifornia 96610

Thesis 166482
N3527 Needels
c.1 Parameterization of terrain in Army combat analysis.